A Two Dimensional Lagrangian Model with Extended Supersymmetry and Infinitely Many Constants of Motion

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Abstract. The reduction of the supersymmetric graded $SU(2|1)/S(U_2 \times U_1)$ σ-model is discussed. If no extra constraint is imposed, one gets a set of two coupled equations (involving two scalar superfields) which generalizes the supersymmetric sine-Gordon equation. It is shown that these equations, which can be derived by a supersymmetric Lagrangian, reproduce in the bosonic limit the reduced version of the $O(4)$ σ-model (Pohlmeyer, Lund Regge, Getmanov model). Moreover the associate linear set and an infinite set of local conservation laws for this new supersymmetric model are exhibited. It turns out that, beyond the spinorial charge which generates the supersymmetry transformations, another unexpected spinorial charge is conserved; then the model appears to be invariant under $N = 2$ extended supersymmetry.

1. Introduction

A supersymmetric generalization of the sine-Gordon equation was proposed some years ago by Di Vecchia, Ferrara and Witten [1]. Later an infinite set of local conservation laws were found for this model [2]; it was shown that they survive also at the quantum level [3] and the S-matrix was calculated [4]. The proof of the classical conservation laws was given [5, 6] by introducing a “Lax set” of linear equations associated to the super sine-Gordon equation.

An interesting (purely bosonic) generalization of the sine-Gordon equation was independently proposed by Pohlmeyer, Lund and Regge [7] who showed its relationship with the $O(4)$ σ-model, and by Getmanov [8]; this model was named the Complex sine-Gordon by Getmanov, but we will call it the GLRP model in order to avoid confusion with a different complexification of the sine-Gordon equation (Complex sine-Gordon II) which was found later [9, 10]. It is very easy to write a supersymmetric generalization of the GLRP model; however for the most obvious supersymmetrization no associate linear set has been found (actually the
attempts to find a supersymmetric extension of the linear set associated to the GLRP led to the discovery of a new model, whose bosonic limit is the Complex sine-Gordon model II [10]).

In [10] and [11] a general scheme to build supersymmetric models endowed with an associate linear set was proposed; it was then possible to write [11] the linear set associated to a new supersymmetric model related to the GLRP model. This model involves 3 coupled scalar superfields, and in the purely bosonic limit gives two sets of decoupled equations: one set reproduces the 2 coupled equations of the GLRP model, the other one is simply the sine-Gordon equation. The sine-Gordon field and the GLRP fields interact through their fermionic counterparts and then the supersymmetric equations cannot be decoupled.

By looking at the GLRP model as a special form of the principal chiral SU(2) model [12], in the present paper we succeed to give a new supersymmetric version, which does not involve any extra field and which is endowed of an associate linear set and of an infinite set of local conservation laws. Interestingly enough, the model shows a larger supersymmetry than the one built in at the beginning: the model is described by superfields of simple supersymmetry, but two spinorial charges appear to be conserved, generating then an extended supersymmetry.

In Sect. 2 we present the Lagrangian and the equations of motion of this model in terms of superfields. In Sects. 3 and 4 we explain how the equations of motion were obtained: after reviewing (Sect. 3) the general method of refs. [10] and [11], in Sect. 4 we apply it to the reduction of the supersymmetric graded SU(2|1) σ-model and we get the linear set associate to our model. In Sect. 5 we write the recursive formula for the infinite set of local conservation laws and briefly discuss the new unexpected supersymmetry. Finally in the Appendix the Lagrangian and the equations of motion are written in terms of ordinary fields.

2. The Lagrangian and the Equations of Motion

The model we wish to discuss is described by the following action

\[ S = i \int d^2x d^2 \theta \left\{ - \frac{i}{2} (D^a \Phi D_a \Phi + \cot^2 \Phi D^a HD_a H) - V(\Phi, H) \right\} \]

where

\[ V(\Phi, H) = m \cos \Phi \cos H, \]

and \( \Phi \) and \( H \) stand for real superfields:

\[ \Phi(x, \theta) = \phi(x) + i \theta^a \psi_a(x) + \frac{i}{2} \theta^a \theta^a F(x), \]

\[ H(x, \theta) = h(x) + i \theta^a \chi_a(x) + \frac{i}{2} \theta^a \theta^a G(x). \]

The corresponding equations of motion are:

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1 We use the notation \( d\theta \) inside the Berezin integrals [13] in order to avoid any confusion with the differentials \( d\theta \) which will come into play in the next sections.