STATISTICAL THEORY OF MICRODEFORMATION OF POLYCRYSTALS IV

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We shall refine the theory of microdeformation of polycrystals proposed earlier. The localization of glide and stress relaxation in plastically deformed grains along with the nonuniformity of the stress state is studied by the construction of a microdeformation curve. Laws of deformation hardening in the domain of microdeformation of polycrystals of solid solutions with fcc and bcc lattices are studied experimentally. Good agreement between theory and experiment is obtained.

In [1-3] we proposed a model for and a theory of microdeformation of polycrystals that takes account of a fundamental property of polycrystals — the nonuniformity of their stress states that is due to different orientations of the grains relative to the axis of the specimen. However, in [1-3] two important factors were left out of account: localization of glide and relaxation of the stress in the grains in which plastic deformation sets in. The present paper is devoted to a consideration of these factors in the theory of microdeformation of polycrystals. A stress—strain diagram is constructed for polycrystals in the microdeformation domain, which is compared with stress—strain curves for polycrystals of solid solutions with bcc and fcc lattices.

In the construction of the stress—strain diagram we employ the following model for the development of a microdeformation in a polycrystal. In the initial stage of the loading all the grains are elastic and the relation between the applied stress \( \sigma \) and the deformation \( \varepsilon \) in the polycrystalline aggregate is described by Hooke’s law. When a specific stress \( \sigma' \) (actual elastic limit) is reached plastic deformation begins in those grains that are under maximum stress and this results in the relaxation of the stress in these grains. But, as the plastically deformed grains are surrounded by elastically deformed grains, the extent of their deformation must be near that of the elastic deformation of the neighboring grains.

Glide bands, originating in the grains under maximum stress, produce stress concentrations in neighboring grains which favor the involvement of these grains in the plastic deformation. The initial involvement of the grains in the plastic deformation due to stress concentrations occurs when the macroscopic elastic limit is reached (that is, when the external stress \( \sigma = \sigma^m \)).

With an increase in \( \sigma \) the number of plastically deformed grains increases and for \( \sigma = \sigma^m \) all the grains in some cross section of the sample become involved in the plastic deformation, i.e., the formation of Luders—Chernov bands ceases. Conclusion of the formation of Luders—Chernov bands corresponds to the transition from micro- to macrodeformation and thus \( \sigma^m \) is the physical yield point of the polycrystal.

Keeping in mind the fact that for \( \sigma < \sigma^m \) there are elastically deformed grains along with the plastically deformed grains, the coefficient of hardening \( d\sigma/d\varepsilon \) can be represented as

\[
\frac{d\sigma}{d\varepsilon} = E n_e + E_p n_p,
\]

where \( n_e \) and \( n_p \) are the relative numbers of elastically and plastically deformed grains respectively, \( E \) is Young’s modulus, and \( E_p \) is the mean coefficient of hardening of the plastically deformed grains, which,

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at the initial instant of microdeformation, is the relaxed modulus of elasticity. This equation is governed by the additivity rule; however, account can be taken of the mutual influence of the grains upon one another through $n_e$ and $n_p$.

In the initial stage of microdeformation the variation in the relative numbers of elastically and plastically deformed grains as the external stress is increased was calculated in [2]. According to [2]

$$n_e = \frac{1}{P-1} \left( \frac{P}{\sigma} \right)^{\frac{1}{\sigma} - 1},$$
$$n_p = \frac{P}{P-1} \left( 1 - \frac{\sigma'}{\sigma} \right),$$

where $P$ is the stress upon attainment of which all the grains become involved in plastic deformation under the condition that this involvement occurs only through the mechanism acting at the first stage of the microdeformation.

In the second state of microdeformation the rate of involvement of the grains in plastic deformation must be higher than it is in the first stage because of the stress concentration produced by the glide bands. The glide band originating in a plastically deformed grain $A$ produces the following stress concentration in a neighboring grain $B$ at a distance $r$ from the end of the glide band in the direction of the axis of tension

$$\left[ \sigma \left( 1 + \nu \right) \left( 1 - D \right) \frac{(\lambda_{111})_A}{E} - \sigma_A \right] \left( \frac{d}{4r} \right)^{1/2},$$

and under an external stress $\sigma$ a stress

$$\sigma_{11} = \sigma \left[ 1 + \nu \left( 1 - D \right) \frac{(\lambda_{111})_A}{E} - \sigma_A \right] \left( \frac{d}{4r} \right)^{1/2} + \frac{\left( 1 + \nu \right) \left( 1 - D \right) (\lambda_{111})_B}{E},$$

acts in the grain $B$ at a distance $r$ from the end of the glide band in the direction of the axis of tension, where $\left( 1 + \nu \right) \left( 1 - D \right) (\lambda_{111})_B/E$ determines the inclination of the direction effective in the grain $B$ from the direction averaged on the basis of the various orientations of the grains, $d$ is the mean diameter of the grains, $(\lambda_{111})_A$ and $(\lambda_{111})_B$ are the deviations of the module-of-elasticity tensor of the grains $A$ and $B$ from the mean value, $\sigma_A = m_A \tau_A$ ($\tau_A$ is the resistance to dislocation motion in the glide band in grain $A$ while $m_A$ is an orientation factor relating to the stress in the glide plane of grain $A$ to the applied stress), $\nu$ is Poisson's coefficient, $D = ((C_{11} - C_{12})/(3C_{11} - 2C_{12} - (1/5)\lambda_6))/(15(C_{44} - (1/5)\lambda_6)(C_{11} - 2C_{44} - (3/5)\lambda_6))$, $\lambda_6 = C_{11} - C_{12} - 2C_{44},$ and $C_{11}$, $C_{12},$ and $C_{44}$ are elastic constants.

Plastic dislocation in grain $B$ starts at that moment when $\sigma_{11}$ attains the value $m_B \tau_T$, where $\tau_T$ is the stress at the commencement of the operation of the dislocation source in one of the glide planes of grain $B$, and $m_B$ is the orientation factor.

Putting $\sigma_{11} = m_B \tau_T$ in (4), we obtain the value of the external tensile stress at which plastic deformation commences in grain $B$, i.e., the micro-yield point of grain $B$:

$$\sigma_B = \frac{m_A \tau_A + m_B \tau_T \left( \frac{4r}{d} \right)^{1/2}}{1 + \left( \frac{\left( 1 + \nu \right) \left( 1 - D \right) (\lambda_{111})_A}{\sigma_A} \right) \left( \frac{d}{4r} \right)^{1/2}}$$

Recalling that multiple glides develop in a polycrystal, we can put $m_A \approx m_B = m_T$, where $m_T$ is Taylor's factor.

When $r << d$ (a condition that is always fulfilled for grain-boundary dislocation sources) $l((\lambda_{111})_B)/(\lambda_{111})_A)$ is $< 1$, as $l((\lambda_{111})_B)/(\lambda_{111})_A)$ is $< 1$. In this case

$$\sigma_B \approx \frac{m_T \left[ \tau_A + \tau_T \left( \frac{4r}{d} \right)^{1/2} \right]}{1 + \nu \left( 1 - D \right) (\lambda_{111})_A}. \frac{(\lambda_{111})_A}{E}$$

From this relation it follows that the distribution of grains according to micro-yield points is determined by the distribution of the quantity $(\lambda_{111})_A$, while the micro-yield points of individual grains range from...