Pole Assignment for Uncertain Systems in a Specified Disk by Output Feedback*

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Abstract. In this paper the problem of pole assignment in a disk by output feedback for continuous- or discrete-time uncertain systems is addressed. A necessary and sufficient condition for quadratic stabilization by output feedback is presented. This condition is expressed in terms of two parameter-dependent Riccati equations whose solutions satisfy two extra conditions. An output stabilization algorithm is derived and a controller formula given.

Key words. Discrete-time uncertain system, Continuous-time uncertain system, Quadratic stabilizability, Lyapunov theory, Pole assignment.

1. Introduction

The problem of pole placement in particular regions of the complex plane for uncertain systems has recently received some attention, see [GGB], [ABG], [HB], [FK], [SKK], and bibliographies therein. This interest can be justified since robust pole placement is a practical way to ensure robust dynamic performance for uncertain systems. In the context of Lyapunov methods, some previous results were developed in [JHW] and [J], where the stability results and pole assignment criteria for uncertain systems have been presented. These methods based on the general theory of matrix root clustering give analysis tools for stability and pole robustness of closed-loop uncertain systems but suffer from a lack of systematic control law determination. In some recent works much effort has been made to derive conditions leading to efficient synthesis procedures for control law design. A sufficient condition for disk location in the case of continuous- or discrete-time systems was established in [FK]. Although systems under consideration are well known, robustness issues are discussed and margins for all poles staying in a disk are evaluated. This condition expressed in terms of a discrete Riccati equation leads to a simple synthesis procedure. The same problem is handled in [GB] where the systems considered are subjected to a norm bounded uncertainty. Using the

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"quadratic d stabilizability" concept which is the counterpart of quadratic stabilizability [B] in the context of pole disk location, a necessary and sufficient condition is derived and it is shown that this condition reduces to the one in [FK] when the system is well known (not uncertain). In [ABG] and [GGB] conditions for placing the poles in several regions (circle, vertical strip, sector,...) are stated for uncertain systems where the parameter uncertainty is defined through convex bounded polytopes. The control law is derived via convex programming. It can be noted that in almost all the papers cited above, the assumption of state availability is made. In practical situations, this assumption is unrealistic and frequently only partial state information through a measured output is available. In these cases, only output feedback control law can be considered. This paper deals with the pole location in a specified disk by output feedback for linear discrete- or continuous-time uncertain systems, the uncertainty being norm bounded. The approach taken follows the ideas of quadratic stabilization. In this direction the quadratic d stabilizability concept introduced in [GB] plays a central role for the results obtained in this paper. A necessary and sufficient condition for quadratic d stabilizability by dynamic output feedback is stated. It is shown that this problem is equivalent to a discrete $H_\infty$ control synthesis problem which can be solved by some standard techniques. The paper is organized as follows. The next section introduces the systems under study and the problem statement. The results relative to quadratic d stabilization by state feedback design are developed in Section 3. The main result with an output d stabilization algorithm is presented in Section 4. We end the paper by some numerical experiments and concluding remarks.

Notations. Throughout the paper the symbols $0$ and $1$ denote respectively the null matrix and the identity matrix of appropriate dimension. $M'$ denotes the transpose (complex conjugate transpose for complex matrices). For symmetric matrices $A$ and $B$, $A < (\leq) B$ means that the matrix $A - B$ is negative definite (semidefinite). $\rho(M)$ denotes the spectral radius of $M$ and $\bar{\sigma}(M) = \rho(M'M)^{1/2}$ is the maximum singular value.

2. Problem Statement

We consider a continuous- or discrete-time system described by
\[
\delta[x(t)] = (A + \Delta A)x(t) + Bu(t),
\]
\[
y(t) = Cx(t),
\]
where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $u(t) \in \mathbb{R}^m$ is the input, and $x(t) \in \mathbb{R}^n$ is the state. $\delta$ is the derivation operator in the continuous-time case (i.e., $\delta[x(t)] = \dot{x}(t)$) and the delay operator for the discrete-time one (i.e., $\delta[x(t)] = x(t + 1)$). $\Delta A$ is the uncertainty of norm bounded type written as
\[
\Delta A = DFE,
\]
where $D \in \mathbb{R}^{n \times r}$, $E \in \mathbb{R}^{l \times n}$ define the structure of the uncertainty and the modeling