The Copeland method*

I.: Relationships and the dictionary

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Summary. A central political and decision science issue is to understand how election outcomes can change with the choice of a procedure or the slate of candidates. These questions are answered for the important Copeland method (CM) where, with a geometric approach, we characterize all relationships among the rankings of positional voting methods and the CM. Then, we characterize all ways CM rankings can vary as candidates enter or leave the election. In this manner new CM strengths and flaws are detected.

Key words: Copeland method, Borda count, Dictionary, Positional Voting, Voting paradoxes.

1.

The Condorcet (or majority) winner [Cn] is the candidate who beats all others (by winning most votes) in pairwise contests. A glaring fault of this widely accepted concept is that it need not exist. Instead, for \( n \geq 5 \) candidates, \( c_1 \) could win all but one pairwise vote while all other candidates lose at least two. Although no one satisfies Condorcet's criterion, \( c_1 \) comes the closest, so it is arguable that she is who the voters want. She does win with Copeland's method (CM) - an important, natural extension of the Condorcet winner [C].

More precisely, in a pairwise competition between \( c_j \) and \( c_k \) let

\[
S_{j,k} = \begin{cases} 
1 & \text{if } c_j \text{ beats } c_k \\
\frac{1}{2} & \text{if } c_j \text{ and } c_k \text{ are tied} \\
0 & \text{if } c_k \text{ beats } c_j 
\end{cases}
\] (1.1)

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The Copeland score for each $c_j$, defined as

$$C(j) = \sum_{k \neq j} s_{j,k},$$

(1.2)

is used to rank the candidates where more is better. Equivalent to these $(1, \frac{1}{2}, 0)$ weights are the $(\frac{1}{3}, 0, 0)$ and $(1, 0, -1)$ choices that we use to simplify proofs. Notice that the CM is the method commonly used to rank hockey and other sport teams.

Trivially, the CM ranking is transitive. (The CM score identifies each candidate with a point on the line, so the transitivity of the election ranking is inherited from the transitivity of points on the line.) Equally as trivial, when a Condorcet winner exists, she is CM top-ranked. (Only the Condorcet winner receives a point from each pairwise contest.) Other CM properties and relevant references are in $[N]$, but, in light of its obvious importance, it is surprising to discover how little is known about this approach. Therefore, a natural objective is to determine the remaining CM properties.

In this and a companion paper $[MS]$, we provide a fairly complete description of the CM properties while emphasizing why they occur. We accomplish this by using a geometric approach ($[SI-4]$) where the geometry helps to discover and verify new conclusions. Because our arguments outline how to use these geometric techniques for other election and choice procedures, this geometric description may be of independent interest.

Here we examine single profile concerns; i.e., the properties, paradoxes, relationships, and perversities of the CM rankings resulting from a fixed profile. We show how to find all possible relationships among the CM and positional rankings of the candidates. (Positional methods extend the plurality vote by giving points to a voter’s lower ranked candidates.) Then we show how to find all ways CM rankings can be related when the set of candidates varies because candidates drop out, new ones are added, or comparisons are desired. In this manner new CM faults are discovered and a large set of profiles is identified for which it is arguable that both the CM and Condorcet winners violate the voters’ true intent.

Multiple profile concerns, addressed in the companion paper $[MS]$, arise by comparing CM outcomes of two or more profiles. To illustrate, the first profile could be the current sincere preferences of the voters. Options for the second profile include strategic action where certain voters change voter type, or where more voters now support a particular candidate, or where a truncated ballot is cast, or where some voters abstain, or where new voters vote, etc. Another choice has each of two profiles representing different subcommittees while a third is the combined group. As such, the $[MS]$ results describe all CM manipulation, consistency, responsiveness, and monotonicity properties.

A flavor of our single profile assertions (where $c_1 \succ c_2$ means that $c_1$ beats $c_2$), comes from profile (i.e., a listing of voters’ preferences) $p^*$ where

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