Note

Conjugate MHD flow past a flat plate

I. Pop, Cluj, Romania, M. Kumari and G. Nath, Bangalore, India

(Received June 29, 1993)

Summary. A boundary layer solution for the conjugate forced convection flow of an electrically conducting fluid over a semi-infinite flat plate in the presence of a transverse magnetic field is presented. The governing nonsimilar partial differential equations are solved numerically using the Keller box method. Values of the temperature profiles of the plate are obtained for various values of the parameters entering the problem and are given in a table and shown on graphs.

1 Introduction

Interest in MHD flow began in 1918, when Hartmann [1] invented the electromagnetic pump. The study of the magnetic field effects on the laminar flow of an incompressible electrically conducting fluid is an important problem that is related to many practical applications, such as MHD power generator and boundary layer flow control.

Historically, Rossow [2] was the first to study the hydrodynamic behaviour of the boundary layer on a semi-infinite flat plate in the presence of a uniform transverse magnetic field. Since then a large amount of literature has been developed on this subject. A review of this topic today can be found in [3]. However, the associated MHD conjugate heat transfer problems have not received any attention due to their complex nature [4]. It is, therefore, the purpose of this note to present a numerical solution of the problem of forced convection layer flow of an electrically conducting, incompressible fluid past a semi-infinite flat plate in the presence of an external magnetic field.

2 Mathematical formulation

Consider the steady boundary layer flow of an electrically conducting fluid past a semi-infinite flat plate of length $L$ and thickness $b$. A magnetic field with a constant magnetic flux density $B_0$ is applied perpendicular to the plate. The external surface of the plate is maintained at a constant temperature $T_0$, which is above the ambient temperature $T_{\infty}$. The physical situation with the coordinate system is depicted in Fig. 1. We assume that the induced magnetic produced by the motion of the electrically conducting fluid is negligible. The assumption is justified since the magnetic Reynolds number is small, which is generally the case in normal aerodynamic applications. Since no external electric field is applied and the effect of polarization of the ionised fluid is negligible, we also can assume that the electric field $E = 0$. Under these assumptions the
The boundary layer equations are

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{1}{\rho} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} (U_\infty - u) \\
\frac{1}{\rho} \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2}.
\end{align*}
\]

Here \( u \) and \( v \) are the components of the fluid velocity in the \( x \) and \( y \) direction, respectively, \( T \) is the fluid temperature, \( \sigma \) is the electrical conductivity of the fluid and \( \rho, \nu \) and \( \alpha \) are respectively the density, kinematic viscosity and thermal diffusivity of the fluid. In Eq. (3), the joule and viscous dissipative heat are assumed to be negligible.

With the assumption that the axial heat conduction in the plate can be neglected, the temperature \( T_s \) in the solid plate is given by the equation

\[
\frac{\partial^2 T_s}{\partial y^2} = 0, \quad 0 \leq x \leq \ell, \quad -b < y \leq 0.
\]

Equations (3) and (4) are coupled by continuity conditions at the solid-fluid interface,

\[
T_s(x, 0^-) = T(x, 0^+) = T_w(x)
\]

and

\[
k_s \frac{\partial T_s}{\partial y} (x, 0^-) = k_f \frac{\partial T}{\partial y} (x, 0^+)
\]

for \( 0 \leq y \leq \ell \). Here \( T_w(x) \) is the unknown temperature at the solid-fluid interface, and will be determined as part of the solution. \( k_s \) and \( k_f \) are the thermal conductivities of the solid and fluid, respectively.

Applying the boundary condition \( T_s = T_0 \) on \( y = -b \), Eqs. (4) and (5.1) give

\[
T_s = T_w(x) + \frac{T_w(x) - T_0}{b} y.
\]