Note

Dynamic response of a cracked beam subject to a moving load

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(Received January 15, 1993)

Summary. The dynamic response of a beam with a single-sided crack subject to a moving load on the opposite side is analyzed using Euler beam theory and the assumed mode method. The beam is modeled as two separate beams divided by the crack. Two different sets of admissible functions which satisfy the respective geometric boundary conditions are assumed for these two fictitious sub-beams. The rotational discontinuity at the crack is modeled by a torsional spring with an equivalent spring constant for the crack. The transverse deflection at the crack is matched by a linear spring of very large stiffness. Results of numerical simulations are presented for various combinations of constant axial velocity of the moving load and the crack size.

1 Introduction

The dynamic behavior of beams may change when cracks begin to emerge in the structure. Knowledge of these changes in the dynamic characteristics is important with respect to crack detection and design of structures. Adams et al. [1] showed that in the axial vibration of a uniform bar, a reduction in stiffness of the bar due to the presence of a crack could be modeled by the introduction of a linear spring. Ju et al. [2] theoretically related the magnitude of the equivalent linear spring constant to the length of the crack in the beam based on fracture mechanics. Several researchers, namely Ju and Mimovich [3], Haisty and Springer [4], and Rizos et al. [5] have experimentally diagnosed the changes in natural frequency produced by cracks and have confirmed the feasibility of modelling a crack by an appropriate combination of linear and torsional springs.

The dynamic response of a beam subject to moving loads has been studied extensively with reference to machining processes and behaviors of railway tracks and bridges. A continuous railway system was modeled in [6] as a multi-span beam. The earliest work on the behavior of a beam subject to a constant moving load was reported by Timoshenko [7]. Exact solutions obtained by solving the partial differential equations using infinite series were presented by Ayre et al. [8] for the dynamic response of a symmetric two-span beam subject to a moving load. Subsequent studies taking into account the effects of moving masses, deflection dependent moving loads, and axial forces were presented by Nelson and Conover [9], Benedetti [10], Steele [11], Florence [12], Katz et al. [13], and Lee [14]. A similar problem of moving loads on beams resting on a tensionless Winkler foundation has been analyzed by Choros and Adams [15], Kenny [16], Kameswara Rao [17], and Weitsman [18]. Numerical results using infinite series were
also presented in [6] for continuous beams with two and six equal spans. The beams considered in all these studies are free of any defects such as cracks.

In the present study, the equations of motion in matrix form for a cracked beam subject to a moving load are formulated using Hamilton's principle and the assumed mode method. The beam is modeled as two separate beams divided by the crack. Two different sets of admissible functions which satisfy the respective geometric boundary conditions are then assumed for these two fictitious sub-beams. The rotational discontinuity at the crack is modeled by a torsional spring with an equivalent spring constant for the crack. The equality for the transverse deflection at the crack is enforced by a linear spring of very large stiffness. Such a model has been successfully applied by the authors [19] for computing the natural frequencies and mode shapes for a beam with single-sided and double-sided cracks with the numerical results in excellent agreement with the reported results using finite element method. The method is modified and expanded in the present paper for computing the dynamic response of a cracked beam subject to a moving load.

2 Theory and formulation

The beam considered is a uniform beam of length $L$ simply-supported at the two ends as shown in Fig. 2. The newly created boundaries at the crack are treated as free ends for the two sub-beams. The global axis for the beam is given by $r$ while $x_1$ and $x_2$ are local axes for the two