Indivisibilities, lotteries, and sunspot equilibria *

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Summary. We analyze economies with indivisible commodities. There are two reasons for doing so. First, we extend and provide some new insights into sunspot equilibrium theory. Finite competitive economies with perfect markets and convex consumption sets do not allow sunspot equilibria; these same economies with nonconvex consumption sets do, and they have several properties that can never arise in convex environments. Second, we provide a reinterpretation of the employment lotteries used in contract theory and in macroeconomic models with indivisible labor. We show how socially optimal employment lotteries can be decentralized as competitive equilibria without lotteries once sunspots are introduced.

1 Introduction

The allocation of resources in the presence of nonconvexities can be an important and complicated problem. Indeed, King Solomon made his name by proposing a mechanism to solve one such problem. In this paper, we analyze economies with indivisible commodities, with two main objectives. First, we extend and provide some new insights into theories of “sunspot equilibria,” theories that examine how extrinsic uncertainty can affect the economy’s resource allocation process and welfare, where uncertainty is said to be extrinsic if it in no way affects the fundamental structure of the economy (that is, its preferences, endowments or technology). Second, we provide a reinterpretation of “employment lotteries,” devices that have been used in contract theory and in macroeconomics to allocate

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resources in economies with indivisible labor. It turns out there is a close connection between sunspots and lotteries; in particular, if competitive markets are designed to accommodate aggregate extrinsic uncertainty, there is no need for agents to use individual randomization devices like lotteries.

In terms of its relationship to the sunspot literature, this work continues the program of characterizing environments in which extrinsic uncertainty plays a role. In (strictly) convex economies, it is well known that: (1) finite economies with complete and unrestricted markets and competitive behavior do not allow equilibria in which sunspots matter; (2) allocations that depend nontrivially on sunspots are never Pareto optimal; (3) equilibria in economies without extrinsic uncertainty always reappear, once extrinsic uncertainty is introduced, as degenerate sunspot equilibria.\(^1\) There has been less work on nonconvex settings. Cass and Polemarchakis (1990) argue that finite, competitive economies with complete, unrestricted markets but nonconvex production sets cannot have nondegenerate sunspot equilibria. Guesnerie and Laffont (1987) and Pietra (1991) consider nonconvex preferences, and do have examples that contain nondegenerate sunspot equilibria and no degenerate equilibria, and also show that these nondegenerate sunspot equilibria can be Pareto optimal.

We study finite competitive economies with complete and unrestricted markets, convex preferences and technology, but nonconvex consumption sets.\(^2\) We show that: (1) these economies can have nondegenerate sunspot equilibria; (2) sunspot equilibria in these economies can be Pareto optimal and can even dominate certainty equilibrium allocations; (3) equilibria in the economy without sunspots do not necessarily reappear as degenerate sunspot equilibria when extrinsic uncertainty is introduced. These contrast with results (1)–(3) above for convex economies (although they are similar to the results for the case of nonconvex preferences). Additionally, in contrast to much of the existing literature, instead of prespecifying the probability distribution of sunspots, we solve for it as part of our equilibrium concept and we analyze the “stability” of sunspot equilibria with respect to generalizations of the exogenous uncertainty and with respect to cooperative agreements among the agents.

These results led us to explore the connection between sunspots and the employment lotteries used in macroeconomics by Rogerson (1984, 1988), Hansen

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\(^1\) See Cass and Shell (1983, 1989). It is also well known that sunspots can matter in some infinite horizon economies, including overlapping generations models (e.g., Shell 1977, Azariadis 1981), and in economies with incomplete markets (Cass 1984), liquidity constraints (Woodford 1986), limited participation (Cass and Shell 1983), or imperfect competition (Peck and Shell 1991). Note that, in this paper, we restrict attention to economies without private information; Cole (1989) provides an example of a finite, convex, private information economy in which extrinsic uncertainty can play a role, although he does not relate his example to the sunspot literature.

\(^2\) To be more precise about what we mean by a finite economy, some of our examples may have a continuum of agents, but the commodity space is always finite dimensional at least before the introduction of sunspots. However, if we were to introduce extrinsic uncertainty by way of a continuous random variable, for example, then the commodity space after the introduction of sunspots would be infinite dimensional. To be more precise about preferences and consumption sets, when we say utility functions are concave we mean they are defined as concave functions on the convex hull of the consumption set.