MAXIMAL ARCS AND DISJOINT MAXIMAL ARCS IN PROJECTIVE PLANES OF ORDER 16

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This paper tabulates the results of a number of computer searches in projective planes of order 16. Maximal arcs of degree 4 are found in all but two of the known planes of order 16 (and their duals). Any such arc yields a resolvable 2-(52,4,1) design that admits at least 52 resolutions. Pairs of disjoint degree 4 maximal arcs are also shown to exist in certain of the planes giving rise to 104-sets of type (4,8).

1 INTRODUCTION

A maximal \(\{q(n - 1) + n; n\}\)-arc in a projective plane of order \(q\) is a subset of \(q(n - 1) + n\) points such that every line meets the set in 0 or \(n\) points for some \(2 \leq n \leq q\). For such a maximal arc \(n\) is called the degree. If \(K\) is a maximal \(\{q(n - 1) + n; n\}\)-arc, the set of lines external to \(K\) is a maximal \(\{q(q - n + 1)/n; q/n\}\)-arc in the dual plane called the dual of \(K\). It follows that a necessary condition for the existence of a maximal \(\{q(n - 1) + n; n\}\)-arc in a projective plane of order \(q\) is that \(n\) divides \(q\).

In [4], W. M. Cherowitzo classified the hyperovals (degree 2 maximal arcs) in the translation planes of order 16 (and hence degree 8 maximal arcs in the dual planes). All translation planes of order 16 were found to contain hyperovals. In [18], Penttila, Royle and Simpson classified by computer all hyperovals in all of the known planes of order 16, and hence classified degree 8 maximal arcs in these planes. In particular Penttila et al showed that hyperovals do not exist in the duals of the Johnson plane, the derived semifield plane, the derived dual of the semifield plane with kernel GF(2), and a certain derived dual of the Hall plane of order 16.

The remaining cases for classification of maximal arcs in these planes of order 16 are then just the degree 4 maximal arcs. In the Desarguesian plane of order 16 the only known degree 4 maximal arcs are those constructed by Denniston in [7]. Up to isomorphism there are two such maximal arcs and they have collineation stabilizers of orders 68 and 408 [11]. In Section
3 of the current paper we report the results of computer searches for degree 4 maximal arcs in the known planes of order 16. Degree 4 maximal arcs are found in all but the derived dual of the semifield plane with kernel $GF(4)$, and a certain derived dual of the Hall plane of order 16, and the duals of these planes.

It is well known that in a projective plane of order $q^2$ the union of the points of $k$ pairwise disjoint degree $q$ maximal arcs gives rise to a $k(q^n - q + n)$-set of type $(kq - q, kq)$, i.e. a set of $k(q^n - q + n)$ points such that every line meets the set in $kq - q$ or $kq$ points (see [8]). In Section 4 we completely classify disjoint pairs of degree 4 maximal arcs in the known planes of order 16 where the maximal arcs of the pair are known to us. It is shown that the Desarguesian plane of order 16, both semifield planes of order 16, the Mathon plane, the Johnson-Walker plane, a certain derived dual of the Hall plane, and all of their duals admit disjoint degree 4 maximal arcs and so 104-sets of type $(4, 8)$.

In the final section of the paper resolvable designs associated with maximal arcs are examined.

2 Known Projective Planes of Order 16

The translation planes of order 16 were classified by Dempwolff and Reifart in [6]. They are: the Desarguesian plane ($PG(2, 16)$); the semifield plane with kernel $GF(4)$ ($Semifield(4)$); the semifield plane with kernel $GF(2)$ ($Semifield(2)$); the Hall plane ($Hall(16)$); the Lorimer-Rahilly plane ($LMRH$); the Johnson-Walker plane ($JOWK$); the derived semifield plane ($DSFP$); and the Dempwolff plane. The Desarguesian plane and the two semifield planes are self dual, while all the others are not.

In addition, the following planes of order 16 (and their duals) are known: two planes that may be obtained by Bose-Barlotti derivation [1] of the Hall plane which we shall denote $BBHall(16)$ and $BB2Hall(16)$ ($BB1Hall(16)$ is self dual while $BB2Hall(16)$ is not); a Bose-Barlotti derivation of the semifield plane with kernel $GF(4)$, which we shall denote $BBSemifield(4)$; the Johnson plane [13]; and the Mathon plane [17]. Of these only $BB1Hall(16)$ is self dual. More details of these planes are available (as are the planes themselves) from the ftp site mentioned at the end of this section.

In [14], N.L. Johnson shows how $DSFP$, $LMRH$ and $JOWK$ may be obtained by derivation of $Semifield(4)$. In [5], Dempwolff and Reifart note that deriving the Dempwolff plane gives $Semifield(2)$. Many of the planes of order 16 are obtainable from one another by derivation. Figure 1 (due to T. Penttila) lists all of the planes of order 16 known to the authors. A box containing a “D” denotes the dual of the plane (for those planes which are not self dual). A line joining two planes denotes that one plane may be obtained from the other by derivation (except in the case of dual Johnson/Mathon in which case a net replacement for six parallel classes of lines is occurring.) There may be many other such relations, these are just the ones known to us.

Gordon Royle has made these planes accessible via the world wide web page from www.cs.uwa.edu.au/~gordon.