CUBIC LINES RELATIVE TO A TRIANGLE

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An extension of the traditional geometry of the triangle is derived through the construction of two 30-points particular cubics. Two generation procedures founded on triangular quadratic transformations and dual associate properties of the two cubics are presented.

1. INTRODUCTION

The existence of a curve $K$ of third degree, relative to any triangle $ABC$, is proved, so that the following thirty significative points of the triangle to be incident with $K$: the vertexes $A$, $B$, $C$; the centroid $G$; the Lemoine point $L$; the midpoints of the sides $A_1, B_1, C_1$; the circumcenter $O$; the orthocenter $H$; the four centers $I, I_1, I_2, I_3$ of the inscribed and ex-inscribed circumferences; the centers of symedians $W, W_a, W_b, W_c$ of the four triangles $II_1I_2$, $II_1I_3$, $II_2I_3$, $II_1I_2I_3$, and their triangular inverse $N, N_a, N_b, N_c$; the midpoints of the altitudes $V_a, V_b, V_c$, and their triangular inverse $U_a, U_b, U_c$; furthermore, two other points $S_0, S_0$ then defined.

To precise the points $U_a, U_b, U_c$ two different elementary constructions are shown: first, as the harmonical conjugate of the diametrally opposite point of any vertex relatively to this vertex and to the intersection of its opposite side and the diameter. Otherwise, $U_a, U_b, U_c$ coincide with the respective intersections of any diameter $OA, OB, OC$ and the corresponding line $V_aG, V_bG, V_cG$. Some more sophisticated constructions will define the four points $N, N_a, N_b, N_c$, as the respective homologous
of the circumcenters of the triangles II'I2, III'I3, III'I3, I1'I2'I3 into a particular quadratic transformation, while the two points S0, S are related to the respective homologous R,S of the two points H,0 through this same transformation.

The corresponding tangents to the cubic K at all above points, mean also significative lines of the triangle ABC.

Furthermore, the cubic K holds associate to a second cubic K', so that a dual correspondence between them is stated.

The former cubic K has been referred long time ago, as a seventeen points cubic (see Ref.[1],p.129), without mention of the associate cubic K'. Besides of the inclusion of other thirteen significative points, also incident with K, i.e., W, Wa, Wb, Wc; their inverse N, Na, Nb, Nc; Ua, Ub, Uc; and S0, S , two particular generations and diverse duality properties of both cubics K, K' are herein discussed.

In fact, the existence and first relation between both cubics K, K' are proved by solving the following problem: Let P, P1, P2, P3 be four harmonically associated points relatively to the triangle ABC (Ref.[2],p.447), then the triangle P1P2P3 being circumscribed to the triangle ABC, as shown in the fig. 1:

![Fig. 1](image)

The curve K containing P has to be determined, so that the perpendiculums on the sides of the triangle P1P2P3 at the