Phase Transition in the 3d Random Field Ising Model

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Abstract. We show that the three-dimensional Ising model coupled to a small random magnetic field is ordered at low temperatures. This means that the lower critical dimension, \( d_1 \) for the theory is \( d_1 \leq 2 \), settling a long controversy on the subject. Our proof is based on an exact Renormalization Group (RG) analysis of the system. This analysis is carried out in the domain wall representation of the system and it is inspired by the scaling arguments of Imry and Ma. The RG acts in the space of Ising models and in the space of random field distributions, driving the former to zero temperature and the latter to zero variance.

1. Introduction

An interesting class of disordered systems is obtained by coupling impurities to the order parameter of a statistical system. This situation may be modelled e.g., by the Ising model (or the \( \phi^4 \)-theory) with a random magnetic field. This random field Ising model (RFIM) describes actual physical systems, such as dilute antiferromagnets in a uniform field [1] and has been used to study, among other things, the effects of impurities on the fluctuations of interfaces [2, 3]. It has also served as a useful playground for various theoretical ideas: the replica trick, [4, 5] dimensional reduction [6, 7, 8, 9] and supersymmetry [10].

As usual, one of the interesting theoretical questions is to determine the upper and lower critical dimensions of the model. The most elegant argument for the upper critical dimension \( d_u \) (i.e., the dimension above which the theory is Gaussian in the infrared) is dimensional reduction, which says that, at long distances and near \( d_u \), the (quenched) correlation functions for the random system behave as those of the corresponding deterministic system, but in two less dimensions. The argument was, basically, to replace the random system by its tree approximation, a stochastic differential (or difference) equation, which, via its representation in

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terms of a SUSY field theory was argued (and proved [11]) to undergo dimensional reduction. While this argument is now known to have many problems, [12, 13], its prediction, \(d_t = 6\), is nevertheless believed to be correct.

Since the lower critical dimension \(d_t\) (above which there is symmetry breaking) of the deterministic system is one, the formal extension [4, 16] of the dimensional reduction argument would yield \(d_t = 3\) for the random system. This result was in contradiction with the earlier scaling arguments of Imry and Ma, [14] which predicted \(d_t = 2\). The controversy was amplified further by the study of the SOS and related interface models for the RFIM. The interface between + and - spins in a system of linear size \(L\) was found to diverge as \(L^{(5-d)/2}\) by the use of the replica trick [4]. This was consistent with the dimensional reduction result, \(d_t = 3\), because, if an interface diverges like its linear size \(L\), one does not expect separation of the system into + and - phases. On the other hand, an Imry–Ma scaling argument [15,2], yielded \(L^{(5-d)/3}\), predicting \(d_t = 2\). The situation was subsequently greatly clarified by two results. First, Fisher, Fröhlich and Spencer [16] and Chalker [7] put the Imry–Ma argument on a much more solid basis and finally a strong case for \(d_t = 2\) was made by Imbrie [18], who proved rigorously, that at zero temperature, the ground state in \(d = 3\) is ordered. However, the case for \(d_t = 3\) could still be made (on the basis of dimensional reduction): the same result for the ground state also holds for the pure system at \(d = 1\), but for \(T \neq 0\) the one-dimensional Ising model is disordered.

In this paper we settle the controversy by proving that, for \(d = 3\) and small disorder, the RFIM is ordered at low temperatures. Thus \(d_t \leq 2\).

Although our result is at variance with the above mentioned predictions of some \(\varepsilon\) expansions (with \(\varepsilon = 5 - d\)), our proof vindicates rather than contradicts the Renormalization Group approach to phase transitions. Indeed, we construct, with no replicas or dimensional interpolations, a straightforward Renormalization Group transformation (suggested by the Imry–Ma argument) under which the system flows towards the zero disorder, zero temperature trivial fixed point.

A combination of our result with the one of [19, 20] on the large disorder regime shows that a phase transition occurs when the variance of the disorder is varied (for \(d \geq 3\) and at low temperatures). Moreover we show that, in the ordered phases, the correlation functions cluster exponentially with probability one.

An extension of our method should prove the following results for interfaces, again at low temperatures and small disorder: for \(d \geq 4\), the interface is essentially flat (like in the deterministic model for \(d \geq 3\)) and for \(d = 3\), the divergence is bounded from above by \(L^{2/3} (= L^{(5-d)/3})\).

We do not, at present, completely prove the conjecture \(d_t = 2\), because we are unable to prove that, in \(d = 2\), an arbitrary small disorder destroys the phase transition, as is suggested by the Imry–Ma argument. However, we have constructed, and explicitly solved, a hierarchical random field model where our Renormalization Group transformation becomes exact [21]. This model has the following properties: for \(d \geq 3\), the system is ordered while, in \(d = 2\), there is no spontaneous magnetization but the correlation functions have a power law decay (such a decay was also found in \(d = 2\) RFIM, at zero temperature [22]). However, we believe that this power law decay is an artifact of the hierarchical model.