TRANSFORMATION FROM PROPER TIME ON EARTH TO
COORDINATE TIME IN SOLAR SYSTEM BARYCENTRIC
SPACE-TIME FRAME OF REFERENCE

Part 1

THEODORE D. MOYER
Jet Propulsion Laboratory, Pasadena, California, 91103, U.S.A.

(Received May 1979; Accepted October, 1979)

Abstract. In order to obtain accurate computed values of Earth-based range and Doppler observables of a
deep space probe, an expression is required for the time difference \( t - \tau \), where \( t \) is coordinate time in the
solar system barycentric space-time frame of reference and \( \tau \) is proper time recorded on a fixed atomic
clock on earth. This paper is part 1 of a two-part article which obtains an expression for \( t - \tau \) which is
suitable for use in obtaining computed values of observations of a spacecraft or celestial body located
anywhere in the solar system. The expression can also be used in the computation of Very Long Baseline
Interferometry data types. Part 1 obtains an expression for \( t - \tau \) which is a function of position and velocity
vectors of the major celestial bodies of the solar system and the atomic clock on Earth which reads \( \tau \). In
Part 2, this expression will be transformed to a function of time and the Earth-fixed coordinates of the
atomic clock.

1. Introduction

This paper is Part 1 of a two-part article which presents the derivation of an
expression for the time difference \( t - \tau \), where \( t \) is coordinate time in the solar system
barycentric space-time frame of reference and \( \tau \) is proper time recorded on a fixed
atomic clock on Earth. Proper time \( \tau \) will refer specifically to International Atomic
Time TAI. The expression is obtained using general relativity; however, to the
accuracy of the retained terms, it is consistent with all viable relativistic theories of
gravitation. The expression for \( t - \tau \) is suitable for use in the computation of
high-accuracy Earth-based range and Doppler observables of a spacecraft or
celestial body located anywhere in the solar system. It can also be used in obtaining
computed values of Very Long Baseline Interferometry data types. The expression
for \( t - \tau \) can be used in orbit determination programs in which the motion of bodies
and light is represented in the solar system barycentric space-time frame of reference
with coordinate time \( t \) as an independent variable.

An expression for \( t' - \tau \), where \( t' \) is coordinate time in the heliocentric space-time
frame of reference, was previously obtained by Moyer (1971). This previous
expression for \( t' - \tau \) was obtained by a straightforward integration of the differential
equation for \( \mathrm{d}\tau / \mathrm{d}t' \). Thomas (1975) has shown that the use of integration by parts
and a first-order expansion of the gravitational potential simplifies the derivation and
provides a clearer understanding of the physical origins of the various terms. The
method of Thomas (1975) is used in Part 1 to obtain an expression for \( t - \tau \), where \( t \) is

* This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory,
California Institute of Technology, under NASA Contract No. NAS 7-100.

Celestial Mechanics 23 (1981) 33–56. 0008-8714/81/0231-0033 $03.60
Copyright © 1981 by D. Reidel Publishing Co., Dordrecht, Holland and Boston, U.S.A.
coordinate time in the solar system barycentric space-time frame of reference. This expression is a function of position and velocity vectors of the major celestial bodies of the solar system and the atomic clock on Earth which reads $\tau$. In Part 2, this expression will be converted to a function of time and the Earth-fixed coordinates of the atomic clock. It includes all of the terms previously obtained by Moyer (1971), with minor changes in the coefficients, and seven new periodic terms.

The following describes briefly the range and Doppler observables of the Deep Space Network (DSN) of the National Aeronautics and Space Administration. An electromagnetic signal is transmitted at a tracking station on Earth, received at the spacecraft and retransmitted, and received at the same or a different tracking station on Earth. The range observable is the round-trip light time of the signal. Doppler observables are proportional to the change in the round-trip light time which occurs during an interval of reception $T_c$ (referred to as the count time) at the receiving station. Range and Doppler observables are referred to as two-way or three-way if the transmitting and receiving stations are the same station or different stations, respectively. Appendix A describes the calculation of the computed values of the range and Doppler observables of the DSN. It is specifically shown how the time difference $t-\tau$ is used in these calculations. Equations are also given for the effects of a term of $t-\tau$ on the computed values of these observables.

The maximum effects of the time difference $t-\tau$ on computed two-way range observables, expressed as equivalent changes in the one-way range $\rho$ from the tracking station to the spacecraft, can be as large as 74.3 m per astronomical unit (AU) of the range $\rho$ plus a range-independent effect of up to 51 m. The maximum effects on computed two-way Doppler observables, expressed as equivalent changes in the one-way range rate $\dot{\rho}$, are $1.72 \times 10^{-3} \text{ m s}^{-1} \text{ AU}^{-1}$ of $\rho$ plus a range-independent effect of up to $42 \times 10^{-3} \text{ m s}^{-1}$. For $\rho < 50$ AU, the maximum effects on three-way data types will not exceed the two-way effects evaluated at $\rho = 50$ AU. However, at low ranges, the three-way effects can be considerably larger than the two-way effects.

Appendix B develops criteria used to determine whether a term of $t-\tau$ must be retained in order to obtain specified accuracies for computed values of two-way range and two-way and three-way Doppler observables. Criteria are not developed for three-way range because it is not a current or currently-planned data type of the DSN. In general, the specified accuracies for computed two-way data types are equal to or greater than the highest obtainable accuracies of the actual observables, for range $\rho$ up to 10 AU. The accuracies of the observables are limited by the frequency stability of the atomic clock at the tracking station (i.e. by the departure of actual atomic time at the tracking station from ideal proper time). The expression for $t-\tau$ is obtained by representing the heliocentric orbit of the Earth–Moon barycenter as an ellipse. The ignored periodic variations in the orbital elements of the Earth–Moon barycenter are the largest error source for computed range observables and one of the largest error sources for computed Doppler observables. Another large error source is the ignored eccentricity of the geocentric lunar orbit.