THE EFFECTS OF DIFFUSE SOLAR RADIATION ON THE MOTION OF AN ARTIFICIAL SATELLITE

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(Received 7 November, 1980; Accepted 5 March, 1982)

Abstract. Previous analyses of the orbits of spherical balloon satellites have attempted to satisfy residuals in observed perturbations in Keplerian elements, assumed to be caused by diffuse radiation pressure, by introducing small variations in \( s \), the parameter representing the reflection characteristics of the satellite's surface. It is difficult to distinguish, however, between those perturbations caused by diffuse radiation and those caused by reflected radiation, as a result of the deformation of the assumed sphere. Following the derivation by Lucas of exact expressions for both incident and reflected radiation forces on a prolate spheroidal satellite, and the subsequent work of Aksnes pertaining to spherical satellites, the theory is extended to include the effects of diffuse radiation whilst at the same time qualifying the assumption that the radiation force acting along the Sun–satellite line can be taken as parallel to the Sun–Earth line.

1. Introduction

In early treatments of the effects of solar radiation pressure (SRP) on the motion of balloon satellites, several authors applied trial and error methods, by incorporating different values for the area-to-mass ratios, to derive results that fitted previously unexplained residuals deduced from the observed perturbations. These methods were fashioned to apply to the satellite Echo 1 in order that the analysis of the semi-major axis might lead to improved values of air density (Shapiro and Jones, 1960). Papers by Musen et al. (1960) and Parkinson et al. (1960) also discuss SRP perturbations, although they do not give general results, but rather the effects on particular satellites. This has also been the case more recently with papers by Fea (1970), Slowey (1974a) and Slowey (1974b).

The time rate of change for orbital elements has been found by employing a vectorial method (Musen, 1960), when the Earth's shadow is neglected. A similar method was utilised by Bryant (1961) to include the effects of this eclipsing of the satellite by the Earth using an iterative technique. This paper required the derived equations to be numerically integrated, and was concerned only with long-period perturbations. A simplified demonstration of how certain resonant conditions of orbital altitudes and inclinations can cause the monotonic build up of effects of SRP was given by Parkinson et al. (1960), while citing Echo 1.

A paper by Fea and Smith (1970) gives the results for the satellite 1963-30D, Dash 2, but again there is no formal presentation of general results. It is of interest, though, to note that after attempting to assign suitable values for the area-to-mass ratio and the reflection coefficient, in order that the best fit could be obtained between the observed and predicted values of eccentricity, a discrepancy was still found. Fea
and Smith explained this as a result of Earth reflected radiation pressure effects.

Now the magnitude of the SRP force per unit satellite mass along the Sun–Earth line, is usually represented in the form (Aksnes, 1976)

\[
\mu F = \frac{A}_{m} \frac{P_0}{r_\odot},
\]

where \( \mu \) is the gravitational constant times the Earth’s mass, \( A/m \) is the cross-sectional area-to-mass ratio of the satellite, \( P_0 \) (approximately \( 4.65 \times 10^{-6} \) N m\(^{-2}\)) is the force per unit area exerted at the Earth by the Sun when its geocentric distance \( r_\odot \) is equal to its mean distance \( a_\odot \), and \( s \) is a constant depending on the reflection characteristics of the satellite’s surface. Thus it is possible to conceal the small effect mentioned above by careful choice of \( sA/m \). However, when this value for \( sA/m \) is used in the development of orbital elements other than eccentricity, the variations between observed and predicted data become rather large. On inspection of the periods where parts of the orbit are in shadow this discrepancy is very marked relative to the periods when the orbit is fully sunlit. It is necessary, therefore, to evaluate and distinguish between the differing effects of Earth reflected radiation and the variation in \( sA/m \) due to rotation of the deformed spheroidal balloon. This conclusion was also arrived at by Aksnes (1976), who found similar discrepancies with Dash 2 while including short period perturbations.

An effect not included by Aksnes is that of a component of radiation pressure acting normally to the Sun–satellite line, a consequence of the departure of the satellite’s shape from that of a sphere. Exact expressions already exist for the radiation forces acting on a prolate spheroid (Lucas, 1974) but include only those effects due to incident and specularly reflected radiation.

The development that follows extends the theory one step further in allowing for the effects of diffuse radiation and, at the same time, examines more closely the assumption that the radiation pressure force can be regarded as acting along the Sun–Earth line.

### 2. The Equations of Motion

In terms of the usual Keplerian elements, Lagrange’s variational equations may be expressed in the form (Cook, 1962; Kozai, 1963; Aksnes, 1976; Moore, 1979)

\[
\frac{da}{dt} = 2na^2(1 - e^2)^{-1/2} F \left[ eS(v) \sin v + T(v) \frac{P}{r} \right],
\]

\[
\frac{de}{dt} = na^2(1 - e^2)^{1/2} F \left\{ S(v) \sin v + T(v) \left[ \cos v + \frac{r}{p} (e + \cos v) \right] \right\},
\]

\[
\frac{di}{dt} = na^2(1 - e^2)^{-1/2} FW \frac{r}{a} \cos u,
\]