EFFECT OF PERTURBATIONS IN CORIOLIS AND CENTRIFUGAL FORCES ON THE STABILITY OF LIBRATION POINTS IN THE RESTRICTED PROBLEM

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Abstract. The location and the stability of the libration points in the restricted problem have been studied when small perturbations $\varepsilon$ and $\varepsilon'$ are given to the Coriolis and the centrifugal forces respectively. It is seen that the points $L_4$ and $L_5$ form nearly equilateral triangles with the primaries and the points $L_1, L_2, L_3$ remain collinear. It is further observed that for the points $L_4$ and $L_5$, the range of stability increases or decreases depending upon whether the point $(\varepsilon, \varepsilon')$ lies in one or the other of the two parts in which the $(\varepsilon, \varepsilon')$ plane is divided by the line $36\varepsilon - 19\varepsilon' = 0$ and the stability of the collinear points is not influenced by the perturbations and they remain unstable.

1. Introduction

It is well known that the restricted problem possesses five libration points. The three collinear points $L_1, L_2, L_3$ are unstable and the two equilateral points $L_4, L_5$ are stable for the mass ratio $\mu$ of the finite bodies less than $\mu_0 = 0.03852 \ldots$ (Szebehely, 1967a). Wintner (1941) showed that the stability of the two equilateral points is due to the existence of the Coriolis terms in the equations of motion. In one of his papers Szebehely (1967b) considered the effect of small disturbances of the Coriolis force on the stability of the libration points keeping the centrifugal force constant. He established that the collinear points remain unstable and for the stability of the triangular points he obtained a relation between the critical value of the mass parameter $\mu_c$ (it is the supremum of the set of all mass ratios $\mu$ for which $L_4, L_5$ are stable) and the change $\varepsilon$ in the coriolis force as

$$\mu_c = \mu_0 + \frac{16\varepsilon}{3\sqrt{69}}$$

establishing that the Coriolis force is a stabilizing force. Subbarao and Sharma (1975) have considered the same problem but with one of the primaries as an oblate spheroid and its equilateral plane coinciding with the plane of motion. The oblateness of the primary resulted in the increase both in the Coriolis force and the centrifugal force. While studying the stability of the triangular points they established that the Coriolis force is not always a stabilizing force.

Since Szebehely’s assertion (viz. Coriolis force is a stabilizing force) depends upon the fact that while changing the Coriolis force, the centrifugal force is kept constant.
whereas in Subbarao and Sharma's paper the centrifugal force is no longer a constant force, we have thought of studying the same problem by taking two parameters $\varepsilon$ and $\varepsilon'$ which represent the perturbations in the Coriolis and the centrifugal forces respectively.

2. The Location of Libration Points

Using non-dimensional variables and a synodic coordinate system $(x, y)$, the equations of motion of the restricted problem are

$$\ddot{x} - 2\dot{y} - x = \frac{\partial F}{\partial x},$$

$$\ddot{y} + 2\dot{x} - y = \frac{\partial F}{\partial y},$$

(1)

where

$$F = \frac{1 - \mu}{r_1} + \frac{\mu}{r_2},$$

$$r_1^2 = (x - \mu)^2 + y^2,$$

$$r_2^2 = (x + 1 - \mu)^2 + y^2$$

(2)

and $\mu$ is the ratio of the mass of the smaller primary to the total mass of the primaries and $0 \leq \mu \leq \frac{1}{2}$. We wish to consider the perturbations in the Coriolis and the centrifugal forces with the help of the parameters $\alpha$ and $\beta$. The unperturbed value of each is unity. Consequently we take the equations of motion as

$$\ddot{x} - 2\alpha \dot{y} - \beta x = \frac{\partial F}{\partial x},$$

$$\ddot{y} + 2\alpha \dot{x} - \beta y = \frac{\partial F}{\partial y}.$$

(3)

Here $\alpha$ and $\beta$ may be taken as

$$\alpha = 1 + \varepsilon, \quad |\varepsilon| \ll 1,$$

$$\beta = 1 + \varepsilon', \quad |\varepsilon'| \ll 1,$$

where $\varepsilon$, $\varepsilon'$ represent the perturbations in the Coriolis and the centrifugal forces respectively.

The Equations (3) can be put in the form

$$\ddot{x} - 2\alpha \dot{y} = \frac{\partial \Omega}{\partial x},$$

$$\ddot{y} + 2\alpha \dot{x} = \frac{\partial \Omega}{\partial y}.$$

(4)