THE THREE-DIMENSIONAL MOTION OF TROJAN ASTEROIDS

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Abstract. The problem is considered within the framework of the elliptic restricted three-body problem. The asymptotic solution is derived by a three-variable expansion procedure. The variables of the expansion represent three time-scales of the asteroids: the revolution around the Sun, the libration around the triangular Lagrangian points $L_4, L_5$, and the motion of the perihelion. The solution is obtained completely in the first order and partly in the second order. The results are given in explicit form for the coordinates as functions of the true anomaly of Jupiter. As an example for the perturbations of the orbital elements the main perturbations of the eccentricity, the perihelion longitude and the longitude of the ascending node are given. Conditions for the libration of the perihelion are also discussed.

1. Introduction

The problem of the Trojan asteroids has always attracted considerable attention in celestial mechanics. The large number of different theories for the Trojans beginning with the work of Brown (1925) to Garfinkel (1977) clearly demonstrate this fact. One of the interesting contributions was made by Kevorkian (1970) using a two-variable expansion procedure (1966). Under the assumptions of the circular restricted three-body problem he pointed out some new features of the motion of these asteroids.

Kevorkian’s method was applied for the Trojan asteroids in the plane elliptic restricted problem of three bodies by this author (1977). It was shown that to derive a solution to second order without secular terms a three-variable asymptotic expansion is needed. This three-variable expansion procedure is applied in this paper for the three-dimensional motion of the Trojan asteroids. The aim of the following considerations is to derive the main perturbations of Jupiter. According to this the problem will be considered under the following assumptions: the asteroids are only influenced by the gravitational forces of the Sun and Jupiter, and the orbit of Jupiter around the Sun is a fixed ellipse.

2. The Equations of the Motion

Let the Cartesian coordinate system $SXYZ$ be specialized as follows: the system is centered at the Sun (see Figure 1), the $SXY$ plane is Jupiter’s orbital plane, the $SX$ axis is directed to the perihelion of Jupiter’s orbit.

To give the position of the asteroid the dimensionless coordinates $r$, $z$ and the angle $\alpha$ will be used. These are related to the rectangular $X$, $Y$, $Z$ coordinates of the asteroid...
asteroid by the relations

\[ X = R_J r \cos (\alpha + \nu) \]

\[ Y = R_J r \sin (\alpha + \nu) \]

\[ Z = R_J z \]

\[ R_J = \frac{a_J (1 - e_j^2)}{1 + e_j \cos \nu}, \]

where \( \nu \) is the true anomaly and \( R_J \) the radius of Jupiter, \( a_J \) the semi-major axis and \( e_j \) the eccentricity of Jupiter's orbit.

Introducing \( \nu \) as an independent variable the following equations of motion may be derived

\[
\frac{d^2r}{dv^2} - r \left( \frac{d\alpha}{dv} \right)^2 - 2r \frac{d\alpha}{dv} \frac{dr}{dv} = \frac{1}{1 + e_j \cos \nu} \left[ r - \frac{1 - \mu}{R_1^3} r + \right. \\
\left. + \mu \left( \frac{\cos \alpha - r}{R_2^3} - \cos \alpha \right) \right] \\
\frac{d}{dv} \left( r^2 \frac{d\alpha}{dv} + r^2 \right) = \frac{\mu r \sin \alpha}{1 + e_j \cos \nu} \left[ 1 - \frac{1}{R_2^3} \right] \\
\frac{d^2z}{dv^2} + z = \frac{z}{1 + e_j \cos \nu} \left[ 1 - \frac{1 - \mu}{R_1^3} - \frac{\mu}{R_2^3} \right],
\]

Fig. 1. The coordinate system.