**Abstract.** Identical equations of motion are shown to emerge for a system of \( n + 1 \) rigid bodies all interconnected by \( n \) points, each of which is common to two bodies, by means of each of the following derivation procedures, all of which employ a kinematical identity developed by Hooker and Margulies: The Hooker-Margulies/Hooker equations; Kane's quasicoordinate formulation of D'Alembert's principle; the combination of Lagrange's generalized coordinate equations and Lagrange's quasicoordinate equations; and the combination of Lagrange's generalized coordinate equations and the vector rotational equation \( \mathbf{M} = \dot{\mathbf{H}} \) applied to the total system and resolved into a vector basis fixed in a reference body of the system. Thus the previously published Hooker-Margulies/Hooker equations are shown to be the natural result of several derivation procedures other than the Newton-Euler method originally used, provided that the central kinematical identity of the original derivation of Hooker and Margulies is employed.

1. **Introduction**

The challenges of spacecraft digital computer simulation have motivated the intensive investigation during the past decade of procedures for formulating equations of motion of complex idealizations of mechanical and electromechanical systems. Because of its relevance to spacecraft modeling and its relative simplicity, particular attention has been given to an idealization consisting of \( n + 1 \) rigid bodies all interconnected by \( n \) points, each of which is common to two bodies; such an idealization is commonly known as a point-connected set of rigid bodies in a topological tree, or a multiple-rigid-body tree, since the network of interconnected bodies is treelike, having no closed loops and no separations between contiguous bodies at their common point.

Certain attractive features of the structure of the equations of motion of a multiple-rigid-body tree were recognized in the course of investigations by Hooker and Margulies [1] and by Roberson and Wittenburg [2]. Stimulated in part by a 1963 paper treating the two-body case [3], Hooker and Margulies developed and published vector-dyadic equations for the multiple-rigid-body tree [1], and Roberson and Wittenburg soon thereafter presented a matrix derivation of equivalent equations [2].

The results in both [1] and [2] were limited in operational value by the retention in the equations of motion of any interbody constraint torques imposed by connections.
permitting fewer than three degrees of relative rotational freedom.* Despite this limitation, the very beautiful but rather imposing equations in [1] and [2] were well received in the aerospace industry, and made the basis for several digital computer numerical integration programs for generic multiple-rigid-body spacecraft simulations (see [4] and [5] for examples). Alternative computer programs were developed based on independent derivations [6 to 8], and multiple-rigid-body simulation programs became standard heavy-duty system evaluation tools in most major aerospace organizations in the period from 1965 to 1970.

In most cases, the equations of motion of that era were written and programmed explicitly but generically, so as to eliminate manual derivations and facilitate numerical computations (without symbolic manipulations on the computer); and all such generic equations retained interbody constraint torques. Russell’s approach [8] was to provide a systematic method for deriving the necessary equations rather than an explicit set of generic equations, but with this sacrifice he gained computational efficiency by avoiding constraint torques. In Russell’s method angular momentum equations were written for nested sets of rigid bodies, and constraint forces and torques were eliminated by judicious substitutions and dot-multiplications; modified angular momentum components about hinge axes serve as variables. All of the referenced programs (and all generic programs of that era known to this writer, with one restricted exception [9]) relied fundamentally upon Newton-Euler formulations of the equations of motion. It was recognized, of course, that with a Lagrangian formulation one could eliminate the troublesome constraint torques, but Lagrange’s equations seemed so cumbersome to work with in the generic multiple-rigid-body case that they were generally rejected.

In 1970 Hooker published a paper [10] which rendered obsolete all of the generic multiple-rigid-body spacecraft simulation programs in which constraint torques remained. Since these equations [10] are a modified form of those appearing in [1], they are called the Hooker-Margulies/Hooker equations (or the HMH equations) in this paper. Their superiority to previous explicit generic equations has made them the basis for most ‘second-generation’ multiple-rigid-body spacecraft simulation computer programs.† (For example, see [12, 13].)

There is, of course, a long and venerable history of development of alternative ways of formulating equations of motion for special classes of idealized mechanical systems. Lagrange’s equations or Hamilton’s canonical equations for conservative, holonomic systems are perhaps the best known and most widely used examples, but the classical literature is laden with other somewhat less famous alternatives, such as the so-called

* In [1] a general method is advanced for the numerical elimination of constraint torques by the computer, but this approach requires digital processing of an augmented system of equations, with consequent cost in computer time.
† The evolution of simulation programs for multiple-rigid-body models of spacecraft has been paralleled by the development of programs for simulating spacecraft idealized as having elastic components, but the latter are excluded from the scope of this paper. See [11] for a more comprehensive treatment.