THE MASSES OF THE PRINCIPAL PLANETS

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Abstract. A set of masses for the principal planets is derived systematically from all available fundamental and independent determinations. In deriving these values an attempt has been made to treat independently those determinations based on differing observational types or analytical methods.

1. Introduction

The planetary masses currently incorporated in the national ephemerides are essentially those deduced by Newcomb (1898a, 1898b). More recently, compilations have been made by Kozlovskaya (1963), Clemence (1964) and Kulikov (1965). Brouwer and Clemence (1961) review past determinations, and also discuss the several methods by which planetary masses can be deduced.

The determination of the mass of a planet, whether derived from its periodic perturbations of an adjacent body, from its effect on the secular motions of other planets, or from measures of the motion of its satellites, may be affected by systematic errors the extent of which are not reflected in the formal mean error given by the investigator. Attempts to evaluate the systematic errors present in the data were nonconclusive, so, while the presence of systematic effects is suspected, there is no known method of correcting for the unknown effect.

In the present study, the various independent determinations of the planetary masses are segregated by observation type and analytical method to form group means. The group means are examined to see if they form an accordant or discordant data set. The group means are then combined to form the final value.

In Tables I through IX are assembled in chronological order all of the published determinations of the masses of the principal planets known to us. Those results which in our judgment are fundamental and independent have been analyzed in groups depending on observation type and analytical method; that is those determinations depending principally on radar measures have been placed together; those resulting from satellite measures are grouped, etc. For each planet the determinations that have been superseded by later investigations utilizing the same data and the same method were omitted from consideration. If previously used data were analyzed again by a significantly different method, the determination was considered independent.

2. Formation of Group Means and Their Errors

The group means were formed by a weighted mean of the closely related but not redundant determinations, the weighting factors being determined usually by the
mean error associated with each value. Where a multitude of measures are available a
method of elimination similar to Taylor et al. (1969) can be used, where any datum
with an error greater than three times that of a similar datum is discarded. In the
case of the planetary masses, there are not sufficient data.

There are two methods to determine the mean error of a weighted mean. The first
method bases the mean error \( e \) of the weighted mean on the mutual discordances of
the individual items with respect to the mean:

\[
e = \sqrt{\frac{\sum w_i r_i^2}{(n - 1) \sum w_i}}
\]  

(1)

where \( w_i \) is the weight of the \( i \)th item, \( r_i \) is the residual of the \( i \)th item with respect
to the mean and \( n \) is the number of individual items entering the mean.

The second method considers the weighted mean to be a simple linear combination
of several items and consequently considers the mean error of the mean to be a linear
combination of the corresponding mean errors of the individual items. The resulting
expression for the mean error of the weighted mean is

\[
e' = \sqrt{\frac{e_0^2}{\sum w_i}}
\]  

(2)

where \( w_i \) is the weight determined from \( w_i = e_0^2/e_i^2 \), \( e_i \) is the mean error of an individual
determination, and \( e_0 \) is an arbitrary, but consistent, quantity. In each case \( e_0 \) was
selected as the largest mean error entering the group mean, so the smallest weight
was unity.

Both methods suffer from the fact that they do not always give a true indication of
the mean error of the mean. For the first method \( e \), consider the case when a mean
value is formed from two items which are very close together, but each having large
mean errors associated with them. Since the deviations from the mean are small, the
resulting mean error will be small, but it will not reflect the large mean errors of the
individual determinations.

In the second method \( e' \), examine the case when a mean value is formed from two
items which are separated by a considerable amount, but each having small mean
errors associated with them. Since the mean errors are small, the mean error of the
mean value will be small, and the deviation of the values making up the mean
will not be reflected in the resulting mean error.

In this investigation, due to the paucity of mass determinations, very few items are
being combined to form a mean value. The character of these data is carefully
examined before choosing the method to form the mean error of the weighted mean.

After forming a weighted group mean, it is frequently the practice in statistics to
discard determinations exceeding three or more times the standard deviation of unit
weight \( \sigma \) as determined by

\[
\sigma = \sqrt{\frac{\sum w_i r_i^2}{(n - 1)}}.
\]  

(3)