TESSERAL RESONANCE EFFECTS ON SATELLITE ORBITS

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Abstract. Resonance effects on satellite orbits due to tesseral harmonics in the potential field have been studied by many authors. Most of these studies have been restricted to nearly circular 24-hour orbits and to the deep resonance regime, where there is exact commensurability between earth rotation and orbit period. Resonance effects have also been noted, however, on eccentric synchronous and subsynchronous orbits and on orbits with far from commensurate periods. These have received much less attention; the object of this paper is to study the whole spectrum of orbits with respect to resonance effects.

1. Introduction

Synchronous (24-hour) satellites are currently of great utility for communication and navigation purposes. However, the synchronous orbit is just a particular case of satellites with periods commensurate with the earth’s rotation; and the special feature of such orbits is that resonances are induced by the longitude-dependent terms in the geopotential. The relevant literature on this subject is extensive [1-11].

The first studies were in connection with the drift of 24-hour satellites under the influence of the dominant longitude-dependent term, namely that associated with the ellipticity of the earth’s equator. A brief description of this phenomenon will point up the basic dynamics and will set the stage for the more advanced development that follows.

Consider a satellite launched into a 24-hour circular equatorial orbit, and let us examine the motion in a frame of reference rotating with the earth. If the equatorial cross-section were circular, the force on the satellite would be central, and in a synchronous circular orbit the satellite would always be at the same geographic longitude (geostationary). In the presence of equatorial ellipticity, however, there is also a net transverse force toward the nearest long axis. From symmetry it is clear that this transverse force must vanish on the extensions of the principal axes of the equatorial ellipse, and that these constitute equilibrium positions.

For a satellite launched in a synchronous orbit at some arbitrary longitude the effective acceleration will be opposite in direction to the force, and the satellite will move toward the minor axis. This is another case of the ‘satellite paradox’ which is well known for drag-perturbed satellites. The drag-perturbed satellites accelerate (because they fall in radially), the synchronous satellites decelerate under the forces $F$ shown in Figure 1a because of an outward movement. The resulting motion will be a long-period (2 years and up) libration of the satellite about the nearest stable

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equilibrium position (Figure 1b). In an inclined orbit the satellite's apparent motion to an earth-fixed observer will be a diurnal figure-eight pattern, symmetrical with respect to the equator; while the node will exhibit the long-period motion in longitude as displayed in Figure 1b.

Fig. 1a. Transverse force ($F_t$) on satellite, stable equilibrium position at $A$ and $C$, unstable ones at $B$ and $D$ (equatorial ellipticity exaggerated). Coordinate system rotating with earth.

Fig. 1b. Paths of geostationary orbits in coordinate system rotating with earth (equatorial ellipticity exaggerated).