PARAMETER OPTIMIZATION AND OPTIMAL ATTITUDE RESPONSE OF A PASSIVE ENVIRONMENTALLY CONTROLLED SPACE SYSTEM

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Abstract. Optimal response criteria are developed for the passive attitude control of a composite spacecraft. The physical structure is uniquely different from multi-body systems in that the components are independently and passively stabilized, here the primary body is gravity oriented while the auxiliary body, through geometry selection, is aerodynamically controlled. The Routh-Hurwitz stability criterion is used to obtain the parameter bounds for stability about the preferred dynamic equilibrium position. Further, the least damped mode concept with respect to the roots of the characteristic equation is used to predict conditions for optimum performance. Numerically generated optimal response curves for a satellite in circular orbit show very rapid damping rates for large disturbances up to ten degrees. Under these conditions, and in the absence of external disturbances, course alignment was reached within several orbits and fine pointing accuracy attainable up to altitudes of 650 kilometers.

Nomenclature

\( A_y, B_y, C_y \) mass moments of inertia about the \( y_1, y_2, y_3 \) axes respectively;
\( A_z, B_z, C_z \) mass moments of inertia about the \( z_1, z_2, z_3 \) axes respectively;
\( C_d \) aerodynamic drag coefficient;
\( F \) first moment of area about the \( z_2 \)-axis;
\([I_y], [I_z]\) inertia matrices of the main and auxiliary bodies respectively;
\( K \) hinge spring constant;
\( K_y \) main body inertia ratio, \((A_y-C_y)/B_y\);
\( K_z \) auxiliary body inertia ratio, \((A_z-C_z)/B_z\);
\( M_A \) aerodynamic torque;
\( O \) center of force;
\( P_z \) aerodynamic parameter;
\( P_c \) damping parameter;
\( P_K \) spring parameter;
\( Q_i \) generalized force associated with the \( i \)th generalized co-ordinate, \( q_i \);
\( R \) distance from the center of force to the system center of mass;
\([T]_{ij}\) transformation matrix between the \( i \)th and \( j \)th co-ordinate sets;
\( T_i \) appropriate element of \([T]_{ij}\);
\( T_y \) kinetic energy of the main body;
\( T_z \) kinetic energy of the auxiliary body;
\( V_c \) satellite orbital velocities (\( V_c=\)reference velocity= circular orbit velocity at \( r_c \));
\( V_{gy} \) gravitational potential of main body;
\( V_{gz} \) gravitational potential of auxiliary body;
\( V_r \) relative velocity of the satellite with respect to the atmosphere;
\( X(x_1, x_2, x_3) \) orbital attitude-reference co-ordinate set;
\( Y(y_1, y_2, y_3) \) body-fixed principal co-ordinates for the main body located at the system center of mass;

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Z(z) body-fixed principal co-ordinates for the auxiliary body located at the system center of mass;
a coefficients defined in Equations (30) and (31);
c hinge damper constant;
d distance from the system center of mass to the auxiliary body center of mass;
e orbit eccentricity;
h, (h0) satellite altitude, (h0 - reference altitude defined in Equation (14));
m, mass of main body;
mz mass of auxiliary body;
m (mz + m);
r location vector of an element of mass measured from 0;
r0 Earth radius;
[r]i transformation matrix for the ith rotation only;
s Laplace variable;
θ angular displacement of R measured from perigee;
ψ, α Euler angles locating the system in pitch relative to X;
λj roots of the characteristic equation;
β system inertia ratio, Kz/Kz;
δ system inertia ratio, Bz/Bz;
ν gravitational field constant;
ω location vector of an elemental mass measured from the system mass center;
ω0, (ω0) atmospheric density at altitude h, (h0);
τ damping index;
ω absolute angular velocity in terms of body-fixed co-ordinates;

Dots and primes represent differentiation with respect to ‘t’ and ‘θ’ respectively.

1. Introduction

For long lifetime satellite missions the use of passive techniques for attitude control are attractive, especially when pointing accuracy requirements are not stringent. Of the many methods proposed, gravity gradiency has attracted the most attention (Klemp-erer and Baker, 1957; Brereton, 1967). While a gravity oriented satellite in circular orbit does have a stable Earth-pointing equilibrium position in the absence of external disturbances, the stability is not asymptotic. In general, with gravity gradient systems, an undamped and often unstable oscillation about the control axis occurs not only from external disturbances but also from orbit eccentricity (Brereton, 1967; Flanagan and Modi, 1971a). A necessary part of the design is, therefore, a means of damping out these oscillations. Among the many passive techniques proposed, the hinged, multi-body systems have received considerable attention in the recent years (Zajac, 1962; Fletcher et al., 1963; Connell, 1969). However, the optimum configurations, when properly tuned and in the absence of external torques, still require damping times in the order of four orbits. This has led to considerable interest in semi-active systems utilizing environmental forces through controlled surfaces on the craft. (Flanagan and Modi, 1971b; Ravindran, 1971). Typically, these systems show good response characteristics but require onboard sensing, logic and energy.

It is well established that the aerodynamic torques on a satellite have a significant effect at altitudes up to several hundred kilometers (Flanagan and Modi, 1972). Based on this fact, in the present paper, a dual stabilized, compound-body system is proposed.