COMMENT ON 'POISSON EQUATIONS OF ROTATIONAL MOTION FOR A RIGID TRIAXIAL BODY WITH APPLICATION TO A TUMBLING ARTIFICIAL SATELLITE' BY LIU AND FITZPATRICK

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Abstract. Liu and Fitzpatrick (1975) discussed the secular and long-periodic behavior of a dynamical system by using improper time-averaged equations. The correct time-averaged equations are given in this note.

In their paper 'Poisson Equations of Rotational Motion for a Rigid Triaxial Body with Application to a Tumbling Artificial Satellite', Liu and Fitzpatrick (1975) discussed the secular and long-period behavior of a dynamical system by using time-averaged equations obtained by direct averaging of the equations of motion. Unfortunately, since the chosen variables were not the action-angle variables of an unperturbed system, their time-averaged equations are incorrect. In order to use direct averaging of the equations of motion, the first-order solutions have to be taken into account. On the other hand, if the averaged Hamiltonian is used, we can get valid time-averaged equations without knowing the first-order solutions. The pitfall in obtaining such time-averaged equations is discussed in detail by Kinoshita (1977).

In the following, we give the proper time-averaged equations. The Hamiltonian of the system is

\[ W = F_0 + F_1 + S, \]

\[ F_0 = \frac{1}{2} \left( \frac{\sin^2 l}{A} + \frac{\cos^2 l}{B} \right) (G^2 - L^2) + \frac{L^2}{2C}, \]

\[ F_1 = \frac{\kappa^2 M \Omega}{r^3} \left[ \frac{2C - A - B}{2} P_2(\sin \beta) + \frac{A - B}{4} P_2^2(\sin \beta) \cos 2\lambda \right], \]

\[ S = -\dot{\Omega} H \cos \lambda + \dot{\Omega} G \sin \lambda \sin \lambda \cos h, \]

where \( A, B, \) and \( C \) are the principal moments of inertia of the satellite, \( G \) is the angular momentum of the rotation, \( L = G \cos \lambda, H = G \cos \lambda, \beta \) is the latitude, \( \lambda \) is the longitude of the Earth with respect to the principal axes of the satellite, and the other variables are defined in Figure 1. The notation used in Liu and Fitzpatrick corresponds with that in this note as follows: \((l, \phi'), (g, \phi_H), (h, \psi_H), (I, \theta'), (I, \theta_H), \) and \((G, \lambda)\) where the first term in the parentheses is ours and the second is theirs. The term \( F_0 \) is the Hamiltonian of torque-free motion; \( F_1 \) gives the gravity-gradient torque due to the central force of the Earth, and \( S \) represents the effect due to the

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motion of the orbital plane, which is caused by oblateness perturbations. In this treatment, $I_0$ and $\dot{\Omega}$ are assumed to be constant. The action-angle variables $\hat{l}$ and $\hat{g}$ corresponding to $l$ and $g$ are given by

$$ l = \delta + \sum_{m=1}^{\infty} \frac{1}{m} \left( -\frac{\varepsilon}{1+\sqrt{1-\varepsilon^2}} \right)^m \sin 2m\delta, \quad (5) $$

$$ \delta = \tilde{l} + 2 \sum_{m=1}^{\infty} \frac{(-q)^m}{m(1+q^{2m})} \sin 2m\tilde{l}, \quad (6) $$

$$ g = \tilde{g} - 2 \sum_{m=1}^{\infty} \frac{(-q)^m}{m(1-q^{2m})} \sin m \frac{\pi c}{K} \sin 2m\tilde{l}, \quad (7) $$

where $q$ is Jacobi's nome,

$$ c = F \left[ \sin^{-1} \sqrt{\frac{1+\varepsilon b}{b(1+\varepsilon)}}, \frac{k'}{k} \right], $$

$$ k'^2 = \frac{2\varepsilon}{1-\varepsilon} \frac{b-1}{1+\varepsilon b}, $$

$$ b = \frac{G^2}{L^2}, $$

$$ \tilde{L} = \sqrt{\left[ 2\alpha_1 - \frac{1}{2} \left( \frac{1}{A} + \frac{1}{B} \right) \right] G^2} D, $$

$$ D = \left[ \frac{1}{C} - \frac{1}{2} \left( \frac{1}{A} + \frac{1}{B} \right) \right]^{-1}, $$

$$ \varepsilon = \frac{1}{2} \left( \frac{1}{B} - \frac{1}{A} \right) D, $$

Fig. 1. The coordinate system as used in this paper.