PERIODIC, CONSECUTIVE-COLLISION ORBITS
IN THE RESTRICTED PROBLEM FOR $\mu \neq \frac{1}{2}$

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Abstract. Sequences of periodic, consecutive-collision orbits in the restricted problem of three bodies have been found for cases where the two primaries are not of equal mass. Each sequence is imbedded in a family of consecutive-collision orbits. There seems to be an infinite number of such families and an infinite number of members in each sequence. Some of the orbits are shown graphically.

1. Introduction

Much of the work in the restricted problem of three bodies has been concerned with the exploration of families of periodic orbits. The extensive list of references is given by Szebehely (1967). Many periodic orbits have been found which contain single collisions – the particle of 'infinitesimal' mass collides with either the primary $m_1$ or the primary $m_2$. Some of these periodic orbits, but only cases with $\mu = \frac{1}{2}$ ($m_1 = m_2$), contain consecutive-collisions – the particle collides first with $m_1$, and then later with $m_2$. Until now, no periodic, consecutive-collision orbits have been found for the case $\mu \neq \frac{1}{2}$ ($m_1 \neq m_2$).

In this paper, families of consecutive-collision orbits are presented, characterized by the initial condition that the particle is ejected from $m_1$ in the positive direction along the $x$-axis. Imbedded in each of these families is a sequence of isolated consecutive-collision orbits which are periodic.

Three families are presented, each of which is seen to originate from the two-body ($\mu = 0$) case. Each family is seen also to approach asymptotically toward a limiting family of periodic, single-collision orbits.

Since there is an infinite number of two-body cases with initial ejection from $m_1$ along the $+x$-axis, there seems to be an infinite number of possible origins for families of consecutive-collision orbits. Furthermore, since the families have asymptotic approaches toward termination, there seems to be an infinite number of periodic, consecutive-collision orbits imbedded in each family.

2. Description of the Orbits

Every orbit shown in this paper has an initial ejection angle from $m_1$, of the value $\Theta_0 = 0^\circ$. The simplest orbit for $\Theta_0 = 0^\circ$ in the two-body case is plotted in the rotating frame in Figure 1. This is simply a rectilinear orbit (in the sidereal frame) where the particle is lofted from $m_1$ out to apocenter and then falls back to collision with the
massless primary $m_2$ in the time, $T=\pi$. The Jacobian constant is $C=1.402$. The collision angle, as illustrated, is $\Theta_c=233^\circ$ in the rotating frame.

The families of consecutive-collision orbits are generated from the $\mu=0$ cases by keeping $\Theta_0=0^\circ$ and by varying $C$ and $\mu$.

![Diagram of consecutive-collision orbit](image)

**Fig. 1.** A consecutive-collision orbit for $\mu=0$. This orbit is the origin of the family, $S_{1,1}$.

### 3. Description of the Families

The families of consecutive-collision orbits found in this study are illustrated by plotting $C$ vs $\mu$ in Figures 2 and 3. Some of the orbits are labeled along the curves by the values of their collision angles, $\Theta_c$.

The orbit shown in Figure 1 is the beginning of the family labeled $S_{1,1}$ (see § 4 for the choice of labeling). As the curve of the family is followed, the point where $\mu=\frac{1}{2}$ is reached with the value of $C=2.433$. More important is the fact that the collision angle, $\Theta_c$, has decreased to the exact value of $180^\circ$. This orbit, shown in Figure 4, is periodic because of the following two facts: (1) the mirror image of an orbit in the restricted problem with respect to the $x$-axis is also a possible trajectory with the particle moving in the opposite sense; i.e., from $m_2$ to $m_1$ (see, e.g., Szebehely, p. 426), (2) both of the angles, $\Theta_0$ and $\Theta_c$, match the corresponding angles of the mirror image.

Since the initial and final conditions of two trajectories have been matched to each other, a periodic orbit is formed. The return trip from $m_2$ to $m_1$ is simply the mirror image of the forward trip from $m_1$ to $m_2$. For the sake of clarity, only the forward trip is shown in Figure 4.