ON THE MOTION OF A SATELLITE IN RESONANCE WITH ITS ROTATING PLANET

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Abstract. The influence of resonance perturbations due to the gravitational field of an oblate planet on its satellite whose motion is commensurable with rotation of the planet has been investigated. It has been shown that in special case of the critical inclination or circular orbit the Lagrange equations can be integrated for all resonance terms simultaneously. The method is applied to the investigation of the motion of the 12-hour communication and navigation satellites of the 'Molniya' and 'Navstar' type. The computations has been performed by the use of four models of the geopotential.

The effects of the tesseral and sectorial harmonics have been investigated by various authors in many papers. The interest in the problem has been stimulated mainly by the practical use for navigation and communication purposes of the artificial Earth's satellites whose motion is commensurable with the rotation of the Earth.

A brief review of the resonance perturbations is given by Dallas (1977). Unfortunately, the latter paper contains a serious error in the analytical solution, as pointed out by Jupp (1979). A survey of the present status of resonance perturbations in artificial satellite motion appears in Garfinkel (1979). This review should be supplemented with a paper by Lidov and Soloviyov (1975) devoted to the study of the qualitative regularities in orbital evolution of the 'Molniya' type satellites by a semianalytical method based on the integration of the satellite motion equations.

The commensurability of the satellite motion and the planet rotation produces as a rule a number of resonance terms which generate small divisors. The study of the perturbations of all resonance terms by analytical approach in spite of efforts of many investigators still remains an unsolved problem. Gedeon and his collaborators (1967, 1969) have made the most significant contribution to the solution of the problem. Gedeon has derived the differential equation of the 'stroboscopic' node longitude, which defines the geographic position of the subsatellite point at the time of intersection of the Earth's equator by a satellite. Numerical integration of the equation with various initial data over the time interval of several thousand days has shown that there is little hope in obtaining a general analytic solution of this equation. Gedeon (1969) pointed out the possibility of existence of the general solution only for some particular cases. These include mainly satellites whose motion is strictly commensurable with the Earth's rotation and whose orbits either are inclined to the equator by \( i = 63.4 \)° (critical inclination) or have the zero eccentricity (\( e = 0 \)). In practice such cases are represented by the 12-hour communication satellites of the
‘Molniya’ type, 24-hour satellites and navigation 12-hour satellites of the ‘Navstar’ type. The present paper is dealing with the analytic solution of Gedeon’s equation for the 12-hour satellites whose orbits are critically inclined and with computation of the first-order perturbations due to resonance tesseral harmonics at the equilibrium point. The secular perturbations due to the harmonics $J_{20}$ and $J_{40}$ are taken into account. The calculations have been carried out for several models of the geopotential on the electronic computer BESM-6.

1. Equation for the Longitude of the Stroboscopic Mean Node

Let a satellite move in the field of the gravitational geopotential which has been given by Kaula (1966) in the following form:

$$V = \frac{\mu}{r} + R = \frac{\mu}{r} + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} V_{lmpq},$$

where

$$V_{lmpq} = \frac{\mu}{r} \left( \frac{a_e}{a} \right)^l F_{lm}(i) G_{ipq}(e) J_{lm} \begin{bmatrix} \cos \psi \\ \sin \psi \end{bmatrix}^{l-m \text{ even}},$$

$$\mu = n^2 a^3, \psi = (l-2p)\omega + (l-2p+q)M + m(\Omega - \theta - \lambda_{lm}),$$

$$J_{lm} \cos m\lambda_{lm} = C_{lm}, \quad J_{lm} \sin m\lambda_{lm} = S_{lm},$$

$a, e, i, \Omega, \omega, M$ are the osculating elements; $F_{lm}(i)$ is the inclination function, $G_{ipq}(e)$ is the eccentricity function, $\mu$ – the gravitational constant, $a_e$ – the mean equatorial radius, $r$ – the radius vector, $\theta$ – the Siderial Time (Angle), $J_{lm}, \lambda_{lm}$ are the coefficient and the longitude of major axes of symmetry of the $(l, m)$ spherical harmonics, respectively; $\psi$ – the trigonometric argument defined above. The satellite motion will be considered in the form of Lagrange’s planetary equations with the disturbing function $R$. After integrating the Lagrange’s equations the analytical expressions of all perturbed elements will have divisors of the following form:

$$\dot{\psi} = (l-2p)\dot{\omega} + (l-2p+q)\dot{M} + m(\dot{\Omega} - \dot{\theta}).$$

Such divisor will be a small quantity if

$$\dot{M} + \dot{\omega} \simeq s(\dot{\theta} - \dot{\Omega}),$$

where $s$ is an integer (the deep resonance). In the case of a small divisor all orbital elements will be affected by large long-periodic perturbations, especially the mean anomaly $M$. Therefore, $\psi$ cannot be regarded as a constant and its changes have to be taken into account. But $\psi$ is an inconvenient variable because each resonance term depends on the four indices $(l, m, p, q)$. In the case of the resonance Gedeon (1967, 1969) uses the longitude of stroboscopic mean node $\lambda_N$ as an independent variable.