AN ENCKE-TYPE SPECIAL PERTURBATION METHOD*

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Abstract. In this paper, a combination analytical-numerical integration method for solving the differential equations of a modified set of Lagrange's planetary equations is described. The integration method is an Encke-type method because it involves integrating the deviations between the actual trajectory and a reference trajectory. The reference trajectory is obtained from an analytical solution containing the dominant secular and periodic effects of the gravitational field of the primary body. A set of nonsingular elements is used so that the method will be valid for all circular and elliptical motions. It is shown that the method is an accurate and efficient means of satellite ephemeris generation.

1. Introduction

Accurate and efficient calculations of the ephemeris of a satellite about its primary body are essential with respect to orbit prediction and correction. An Encke-type numerical-integration method which is computationally more accurate and efficient than a classical Encke or Cowell technique is described in this paper. The algorithm described integrates the deviations of the true state from a base or reference state which is obtained from a general perturbation solution. The reference solution contains the dominant secular and periodic perturbations caused by the primary body.

Previous extensions of the classical Encke method have been made by Kyner and Bennett [1] who showed that the time to rectification and the accuracy of the classical Encke method are increased substantially by including first-order secular effects of oblateness in the base solution. This study differs from the work of Kyner and Bennett in that the base solution that is used includes both the periodic and secular effects of any desired harmonic in the disturbing function. Because the integration step size is determined primarily by short-period variations, this base solution should allow a much larger step size than one containing only secular variations. In addition, Kyner and Bennett used Cartesian coordinates; a set of orbit elements is used in this study.

An extension of the classical Encke method for the n-body problem has been made by Stumpff and Weiss [2] who present the derivation of an n-body reference orbit which differs from the actual orbit by terms of the fourth order.

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2. Theory

The theory for the modified Encke method is summarized as follows. The general form of Lagrange's planetary equations is

$$\dot{\eta}_i(t) = f_i(\eta_j, \varepsilon, t),$$  \hspace{1cm} (1)

where $j=1...6$, $i=1...6$, $\eta_j$ are the orbital elements, $\varepsilon$ is a small parameter proportional to the magnitude of the disturbing force, and $t$ is time. When the disturbing force has a gravitational origin, an approximate analytical solution for $\eta(t)$ exists of the form $[3, 4]$,

$$\eta(t) = \eta_0 + g(\bar{\eta}, \dot{\eta}_s, t),$$  \hspace{1cm} (2)

where $\eta_a$ is the analytical value, $\bar{\eta}$ is the mean value, $\eta_0$ is the epoch value, and $\dot{\eta}_s$ is the secular rate.

Because the solution for the deviations between the true and analytical values is desired, let

$$\Delta \eta = \eta_T - \eta_a$$ \hspace{1cm} (3)

and then

$$\Delta \dot{\eta} = \dot{\eta}_T - \dot{\eta}_a,$$ \hspace{1cm} (4)

where $\eta_T$ is the true value. Equation (4) is the system to be integrated, subject to the initial conditions that $\Delta \eta(t_0) = 0$. The analytical values and their derivatives are evaluated by using Equation (2). The quantity $\dot{\eta}_T$ is obtained from Lagrange's planetary equations. After $\Delta \eta$ has been determined, the value of $\eta_T$ is obtained from Equation (3).

3. Equations of Motion

The analytical solution used for this investigation is presented in Reference [3]. This solution was chosen because $\eta_a$ and $\dot{\eta}_a$ can be evaluated simultaneously. In addition, the third-body effect may be included in the reference solution. The solution will be summarized in the following discussion; Reference [3] may be consulted for more detail.

The nonsingular elements are as follows: $h = \sin I \sin \Omega$, $k = \sin I \cos \Omega$, $A = e \sin(\omega + \alpha \Omega)$, $B = e \cos(\omega + \alpha \Omega)$, and $\delta = \alpha \Omega + \omega + M$; where $I$ is the inclination, $\Omega$ is the longitude of the ascending node, $e$ is the eccentricity, $\omega$ is the argument of pericenter, and $M$ is the mean anomaly. The sixth element $a$ is the semimajor axis. The quantity $a$ has the value $\pm 1$, depending on the inclination, that is, $\alpha = -1$ for $180^\circ \geq I > 175^\circ$, or $\alpha = +1$ for $175^\circ \geq I \geq 0^\circ$. The associated differential equations are

$$\frac{da}{dt} = \left( \frac{2}{na} \right) \left( \frac{\partial R}{\partial M} \right)$$ \hspace{1cm} (5a)

$$\frac{dh}{dt} = k \frac{d\Omega}{dt} + \cos I \sin \Omega \frac{dI}{dt}$$ \hspace{1cm} (5b)