FLAG TRANSITIVE PLANES OF DIMENSION FOUR OVER GF(3)

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Two non desarguesian flag transitive planes of order $3^4$ whose Kernel is GF(3) are constructed. These planes are distinct from the planes of the same order contained in the class constructed by Narayana Rao, M.L. (Proceedings of American Mathematical Society 39 (1973) 51-56) and Ebert, G.L. and Baker, R. (Enumeration of two dimensional Flag-Transitive planes, Algebras, Groups and Geometries 3 (1985) 248-257). The Flag Transitive group modulo the scalar collineations of these planes is generated by two elements and is of order 328.

1. INTRODUCTION

An affine plane is said to be flag transitive if the plane admits a collineation group which is transitive on the incident point line pairs (flags) of $\pi$. Wagner [14] has shown that a finite flag transitive plane is a translation plane so that its order is a prime power. Foulser [3] has determined all flag transitive groups of finite affine planes. Besides desarguesian planes, the following are the finite flag-transitive planes of odd order reported in the literature till this date.

i) The two flag transitive planes of Foulser of order 25,[4].
ii) The nearfield plane of order 9.

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iii) The flag transitive plane of Narayana Rao of order 49 [7].

iv) A class of flag transitive planes of square order constructed by Narayana Rao [8].

v) The two dimensional flag transitive planes of Ebert [2].

vi) Hering’s flag transitive plane of order 27 [5].

vii) The flag transitive plane of Narayana Rao and Kuppuswamy Rao of order 27 [9].

viii) The flag transitive plane of Narayana Rao, Kuppuswamy Rao and Satyanarayana of Order 27 [12].

ix) The flag transitive plane of Narayana Rao and Kuppuswamy Rao of order 125 [10].

Complete collineation groups of the flag transitive planes mentioned in (i), (vi), (vii) and (viii) have been determined in [3], [11], [13] and [12] respectively. All the planes mentioned above are non desarguesian and are such that the square ordered planes are of dimension of 2 over their Kernel and the cube ordered planes are of dimension 3 over their Kernel. The aim of this paper is to construct a non desarguesian flag transitive plane of order 34 which is a proper four dimensional plane over GF(3). We are indebted to Ostrom, T.G. who suggested to us in a private communication to construct finite translation planes which are (proper) four dimensional over their Kernels. The planes constructed in this paper have flag transitive groups generated by two mappings $x \rightarrow x \alpha^{60}$ and $x \rightarrow x^3 \alpha^{809}$ or $x \rightarrow x^3 \alpha^{849}$ where $\alpha$ is a primitive element generating GF(3) as an eight dimensional vector space over GF(3). This result tallys with the structure of flag transitive groups determined by Foulser [3].

2. SOME PRELIMINARY RESULTS

Let $V$ be a vector space of dimension 8 over GF(3). Let $V_i$, $0 < i < 81$, be a set of 4 dimensional vector spaces of $V$ over