V. I. FERRONSKY
International Atomic Energy Agency, Wagramerstr. 5, P. O. Box 100,
A-1400 Vienna, Austria

and

S. A. DENISIK and S. V. FERRONSKY
Water Problems Institute of the Academy of Sciences of the USSR,
13/3 Sadovaya-Chernogryazskaya, Moscow K.64, U.S.S.R.

(Received 30 September; accepted 17 October, 1983)

ABSTRACT. Generalized Jacobi's equation is derived by introducing the friction force into the equations of motion of mass points constituting the system.

The exact solution of the equation of virial oscillations of celestial bodies written for non-conservative systems is obtained using non-linear time scale in the course of the change of variables for a particular friction force law.

The nature of the undamped virial oscillations of celestial bodies is though to be related to the system unstability near the state determined by the virial theorem. Thus, the friction force changes its sign near the unstable equilibrium state and due to dissipation of energy during evolution of the system the undamped virial oscillations can be described as self-exited oscillations.

1. INTRODUCTION

In our work (Ferronsky et al., 1979) the solution of equation of virial oscillations for non-conservative systems was obtained which enabled us to study analytically the problem of evolution of celestial bodies. But in the framework of considered formulation of the problem the account of the energy dissipation was carried out on the basis of unphysical model by assuming a formal energy dissipation function.

The solution of the problem can be obtained using more physical supposition by taking into account the friction forces acting at the mass points consisting the system. In this case the generalized Jacobi's equation should be derived by introducing the friction forces into the differential equations of motion.

It will be shown below that the obtained equation of virial oscillations describing evolution of the non-conservative system writes as follows:
\[
\phi + \dot{\phi}F(t, \phi, \dot{\phi}) - \frac{B}{\sqrt{\phi}} + A = 0,
\]

(1)

where \( \phi, \dot{\phi}, \ddot{\phi} \) are Jacobi's function and its first and second derivatives with respect to time \( t \); \( A \) and \( B \) are constants; \( F(t, \phi, \dot{\phi}) \) is a given function depending on the accepted law of friction forces.

Equation (1) can be solved using well-known perturbation methods with all restrictions and difficulties following from them. But in some cases (e.g. when \( F(t, \phi, \dot{\phi}) = k/\sqrt{\phi} \)) this equation can be solved exactly. The exact solution is obtained by introducing a new independent parameter \( \lambda \) and writing two linear differential equations of the second order for functions \( \phi(\lambda) \) and \( t(\lambda) \) in the form

\[
(\sqrt{\phi})'' + \sqrt{\frac{2}{A}} k(\sqrt{\phi})' + \sqrt{\phi} = \frac{B}{A},
\]

(2)

\[
t'' + \sqrt{\frac{2}{A}} kt' + t = \frac{4B}{(2A)^{3/2}} \lambda,
\]

(3)

where prims denote differentiation with respect to \( \lambda \); \( k \) is a constant.

The solution of the system of Equations (2) and (3) can be easily found. The partial solution of this system, having only two integration constants represent general solution of the non-linear equation (1) at \( F(t, \phi, \dot{\phi}) = k/\sqrt{\phi} \).

It is worth to note that Equation (3) expresses the non-uniformity of time flow since according to the Newtonian mechanics \( t'' \) should be equal to zero.

In contrast to the methods used earlier for solution of equations analogous to the Equation (1), here the elimination of parameter \( \lambda \) can be carried out from the solutions of the system of linear equations (2) and (3) being algebraic expressions. Thus the obtained solution can be considered as exact. This circumstance allows us to recommend the method for solution of other problems of celestial mechanics, related to calculation of the orbits of motion of celestial bodies. An advantage of the method consists in elimination of mistakes appearing in the course of numerical calculation of the differential equations of motion due to accumulation of the mistakes of approximation.

2. DERIVATION OF THE EQUATION OF VIRIAL OSCILLATIONS FOR NONCONSERVATIVE SYSTEMS

Let us consider the system of \( N \) mass points the motion of which is determined by the force of their mutual gravitation attraction and the friction force. It is well known that the friction force always appears in the course of evolution of any natural system. It is also known that there is no uni-