Abstract. A wide variety of equal-mass stellar triple systems has been numerically integrated in order to establish factors pertinent to stability. The significant parameters appear to be whether the relative revolution is direct or retrograde, and the ratio of the periastron distance in the outer orbit to the semi-major axis of the inner orbit. For stability, this ratio must be at least 3.5 for direct orbits and at least 2.75 for retrograde orbits.

1. Introduction

There are in the universe, besides the well-known clusters and associations, many multiple star systems, the known systems containing anywhere from two to more than a dozen components. There has been speculation that almost all stars are grouped in such systems, with the number of systems with \( N \) components decreasing with \( N \). (The solar system has often been described as a many-component system, but planetary systems are dynamically different in their qualitative aspects from stellar systems and so should not be considered as such.) Further, multiple stars are found to have a hierarchical arrangement (Evans, 1968), such that, observationally, their motions can be described by series of two-body motions. The trapezium systems, for which the apparent separations are all of the same order, are either young and unstable, or the hierarchical distribution is masked by projection (Sharpless, 1966).

Of particular interest dynamically are the triple stars. Even though there are many known triple stars, there are only a few that have the right combination of separations and periods to make it possible to determine both the orbit of the close pair (the inner orbit), and that of the distant component with respect to the close pair (the outer orbit). Table I lists some of the characteristics of the six systems with both orbits determined. The orbital elements are taken from the Third Catalogue of Orbits of Visual Binary Stars, by Finsen and Worley (1970). Also given are the ratio of the outer

<table>
<thead>
<tr>
<th>System</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( a_2/a_1 )</th>
<th>( q_2/a_1 )</th>
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</thead>
<tbody>
<tr>
<td>(-30,^\circ , 529)</td>
<td>0'17</td>
<td>1'44</td>
<td>0.32</td>
<td>0.22</td>
<td>8.4</td>
<td>6.6</td>
</tr>
<tr>
<td>(\zeta\ \text{Cnc} )</td>
<td>0.88</td>
<td>7.96</td>
<td>0.32</td>
<td>0.26</td>
<td>9.0</td>
<td>6.7</td>
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<tr>
<td>ADS 3358</td>
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<td>1.38</td>
<td>0.54</td>
<td>0.30</td>
<td>7.7</td>
<td>5.4</td>
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<tr>
<td>(e\ \text{Hyd} )</td>
<td>0.24</td>
<td>4.54</td>
<td>0.67</td>
<td>0.29</td>
<td>19.1</td>
<td>13.5</td>
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<tr>
<td>(\xi\ \text{U Maj} )</td>
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<td>2.53</td>
<td>0.41</td>
<td>0.56</td>
<td>12.4</td>
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<tr>
<td>(\zeta\ \text{Aqr} )</td>
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<td>5.06</td>
<td>0.50</td>
<td>0.26</td>
<td>13.0</td>
<td>9.6</td>
</tr>
</tbody>
</table>

TABLE I

Completely determined triple star systems

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to inner semi-major axis, $a_2/a_1$, and the ratio of the outer periastron distance to inner semi-major axis, $q_z/a_1$. Note that all values of $a_2/a_1$ are around 10, the smallest being 7.7. There are many systems with larger values, but they haven’t been observed sufficiently to enable their orbits to be determined, since the period of the outer orbit is too large. However, it is generally assumed that the observational value of about 10 represents a genuine lower limit for this ratio, and it would be of interest to know if this limit reflects some property of the formation process of multiple stars, or whether it merely reflects a limit on the dynamical stability of such systems. This is the question towards which the present study is directed.

The question can be reformulated from a somewhat different point of view. The stellar three-body problem is defined as the problem of the motion of three bodies of finite, comparable masses moving under their mutual gravitational influences, subject only to the restriction that the separation of two of the bodies is small compared to the separation of either from the third. This motion can be considered as the combination of perturbed two-body motion of the close pair plus perturbed two-body motion of the third body with respect to the center of mass of the close pair. Provided the ratio of outer to inner semi-major axis is ‘large enough’, the motion can be shown to be stable, in the sense that there are no secular terms in the semi-major axes (Harrington 1968, 1969). The question then arises as to what is meant by ‘large enough’, and in particular if it is smaller than the approximate 10 implied observationally.

2. Formulation

Consider three finite masses, $m_0$, $m_1$, $m_2$, moving such that $m_0$ and $m_1$ are the close pair. Note that, though point masses are assumed, it will not be overlooked that stars both have finite size and can lose spherical symmetry by tidal interaction if they get too close. By taking into account the conservation of linear momentum, the motion of the system can be described by considering the Jacobian system of variables, the vector $r_1$ from $m_0$ to $m_1$, and the vector $r_2$ from the barycenter of $m_0$ and $m_1$ to $m_2$. The equations of motion then become the following:

$$\begin{align*}
\ddot{r}_1 &= - G (m_0 + m_1) \left[ \frac{r_1}{|r_1|^3} + \frac{m_2}{m_0 + m_1} \frac{r_{02}}{|r_{02}|^3} - \frac{m_2}{m_0 + m_1} \frac{r_{12}}{|r_{12}|^3} \right] \\
\ddot{r}_2 &= - G \frac{(m_0 + m_1 + m_2)}{(m_0 + m_1)} \left[ \frac{m_0 r_{02}}{|r_{02}|^3} + \frac{m_1 r_{12}}{|r_{12}|^3} \right].
\end{align*}$$

(1)

The vector $r_{02} = r_2 + m_1/(m_0 + m_1) r_1$ is the vector from $m_0$ to $m_2$, and $r_{12} = r_2 + m_0 (m_0 + m_1) r_1$ is the vector from $m_1$ to $m_2$. The Gaussian constant, $G$, equals $4\pi^2$ if the units are astronomical units, solar masses, and years.

Equations (1) represent six second-order differential equations describing the motion of the system. These equations are more complex than the nine resulting second-order equations if all separation vectors between the components are considered. Nevertheless, the motions of the vectors $r_1$ and $r_2$ are the ones approximated by two-body