ORBITAL CHARACTERISTICS OF DYNAMICAL MODELS OF ELLIPTICAL GALAXIES

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Abstract. We use three different dynamical models for describing the motion in the meridian plane of an elliptical galaxy. Orbital characteristics of two polynomial models are compared with those given by a logarithmic potential for a fixed axial ratio of the equipotential surfaces. We also study the effects of an asymmetric perturbation caused by a companion galaxy on the orbital behaviour of the above models. Finally we present some theoretical arguments in order to support the numerical results.

1. Introduction

In order to study the properties of stellar orbits in elliptical galaxies Richstone (1982) and Katz and Richstone (1984) used a logarithmic potential given in spherical coordinates, with measured from the symmetry axis, by the equation

\[ V_R = \frac{1}{2} \ln \{r^2 - 2(1 - q^{-2}) \cos^2 \theta \} \]

where the flattening parameter \( q \) can vary from \( 1/\sqrt{2} \) to 1.

On the other hand some investigators (see for instance, Caranicolas and Barbanis, 1982; De Zeeuw and Merritt, 1983; Davoust, 1983) have adopted bisymmetrical fourth order potentials made up of harmonic oscillators for the study of stellar motion in elliptical galaxies.

Caranicolas and Barbanis (1982, hereafter paper I) used the potential

\[ V = \frac{1}{2}(Ax^2 + By^2) - \varepsilon(x^2 + \alpha y^2)^2, \]

where \( A \) is equal or near \( B \) and \( 0 < \alpha < 1 \) in order to describe the motion on the plane of symmetry of a nearly axisymmetric galaxy. In this case Equation (2) is more or less an arbitrary choice for the description of a nearly axisymmetric stellar system. We could also have used a rational or an exponential potential for the same purpose. Furthermore the results given by the simple potential of the form (2) are valid only near an equilibrium point (see also Mayer and Martinet, 1973).

For these reasons we find it of interest to compare the orbital characteristics of three different dynamical models describing the motion on the meridian plane of an axisymmetric elliptical galaxy. The first potential is that of an anharmonic oscillator with equal frequencies

\[ V_A = \frac{1}{2}(x^2 + y^2) - \varepsilon(x^2 + \alpha y^2)^2, \]

where \( \varepsilon \) is not considered as a parameter but as a small quantity that introduces the non-linearity. The second model is the polynomial potential

\[ V_P = \frac{1}{2}(x^2 + y^2) - \varepsilon(x^2 + \alpha y^2)^2, \]
\[ V_p = (x^2 + \alpha y^2)^2 \]  
\[ V_R = \frac{1}{2} \ln (x^2 + \alpha y^2)^2 \]

and the third is the logarithmic potential (1).

Writing equation (1) in rectangular coordinates we find
\[ V_R = \frac{1}{2} \ln (x^2 + \alpha y^2)^2 \]

where \( 1 \leq \alpha \leq 2 \). The same values of \( \alpha \) are adopted for the models given by equations (3) and (4) (hereafter models A and P respectively). The values of the energy constant \( h \) is taken equal to 0.5 and 1 for the models A and P respectively while for Richstone potential (5) (hereafter model R) is taken equal to zero. Our choice for \( h \) was not entirely arbitrary. We wanted to have galaxies of about the same size. This makes the comparison of the orbital characteristics for the three models much more easy and interesting.

We start our investigation with a detailed examination of the zero velocity curves for different values of the flattening parameter \( \alpha \) and the calculation of the energy of escape for each model. Furthermore, in the same second Section, we present the form of the zero velocity curves in the case where the three galactic models are subjects to an asymmetric perturbation caused by a companion galaxy. Then in Section 3 we compare the orbital characteristics of the three models in both unperturbed and perturbed case by considering a surface of section. Some features of the model A and P are explained by means of a second formal integral. Finally, Section 4 contains a discussion and the conclusions of this work.

2. The Zero Velocity Curves

The Hamiltonian to the potential \( A, P \) and \( R \) is given by the equation
\[ H = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + V_M = h \]

where \( V_M \) is replaced by \( V_A, V_p, \) or \( V_R \) respectively. The corresponding zero velocity curves (ZVC) are given by the equation
\[ h - V_M = 0. \]

In order to study the ZVC we follow the method described in an earlier paper by Caranicolas and Varvoglis (1984). This analysis gives an energy of escape
\[ h_{esc} = 1/(16\alpha x^2) \]

for model A while for models P and R the ZVC are always closed curves (ellipses).

In order to describe the perturbation caused by a companion galaxy we may add in potential \( V_M \) the linear term \(-\lambda x(\dot{x} > 0)\). Then the general form of equation giving the ZVC becomes
\[ h - V_M + \lambda x = 0. \]

In the case of model A the same analysis gives one saddle point on the \( x \)-axis while it is difficult to express analytically \( \lambda_{esc} \) because one has to solve cubic