AN ANALYTIC SATELLITE THEORY USING GRAVITY AND A
DYNAMIC ATMOSPHERE

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Abstract. An analytical solution is given for the motion of an artificial Earth satellite under the combined influences of gravity and atmospheric drag. The gravitational effects of the zonal harmonics $J_2$, $J_3$, and $J_4$ are included, and the drag effects of any arbitrary dynamic atmosphere are included. By a dynamic atmosphere, we mean any of the modern empirical models which use various observed solar and geophysical parameters as inputs to produce a dynamically varying atmosphere model. The subtleties of using such an atmosphere model with an analytic theory are explored, and real world data is used to determine the optimum implementation. Performance is measured by predictions against real world satellites. As a point of reference, predictions against a special perturbations model are also given.

1. Introduction

The motion of a low altitude Earth orbiting satellite is influenced primarily by the Earth’s gravitational field and the atmospheric drag. To predict this motion, one must select a mathematical representation for these forces and must decide on a mathematical solution technique for integrating the resulting differential equations of motion. Often the decision on the first question will limit the options on the second question and vice versa. We first address the options for the mathematical models.

Many quite extensive models for the gravitational field are available. However, all have the same basic mathematical structure and differ only in values of their empirically determined coefficients. Thus, that portion of the differential equations arising from various gravitational fields will differ only in the values of the constant coefficients. On the other hand, atmospheric models may be analytical or tabular or a combination. Most analytical models are based on a spherically symmetric exponential or power function density representation. Modifications to these functions can be made to approximate atmospheric oblateness and diurnal bulge but analytical inclusion of the dynamic variation due to solar activity has not yet been accomplished. Tabular and combination models are empirically determined, reflect the dynamics of the atmosphere due to solar influences, and generally provide more accurate density values. We will refer to such a model as a dynamic atmosphere model.

The options for mathematical solution techniques can be classified as numerical, semianalytical, and analytical. A numerical method applied numerical integration to the osculating differential equations to obtain the state at a later time. A semianalytical method uses analytical equations to transform from the osculating to a mean set of differential equations. Since the mean equations are more slowly varying

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than the osculating ones, a numerical integration can be performed using a much larger step size than usual to obtain the mean state at a later time. The osculating state is obtained through the inverse analytical transformation equations. Generally, an analytical method uses analytical transformation equations to transform from the osculating to a mean set of differential equations. The equations are integrated analytically and can readily be used to predict the mean state at a later time. The osculating state is obtained through the inverse analytical transformation equations. Both analytical and semianalytical methods use an analytical transformation to produce mean, slowly varying differential equations. However, the major distinction lies in the fact that semianalytical methods numerically integrate these equations whereas analytical methods provide a totally analytical solution to these differential equations. Consequently, an analytical method goes through an initialization section only one time where a semianalytical method must go through a reinitialization at each integration step. For a very thorough discussion of the state-of-the-art in analytical and semianalytical modeling of the gravity-drag problem, see Liu (1983).

For those who, because of program size, runtime, or other limitations, must use an analytical method, it would be desirable to develop a framework for an analytical theory which models gravitation and the effects of a dynamic atmosphere, yet whose algebraic formulation is independent of the density model chosen. In this way we have the flexibility to select both the geopotential coefficients and the density model for a given analytical prediction.

In a recent paper, Hoots (1982) has presented such a model. The performance of the model versus reference orbits simulated with numerical integration was reported. Over the last two years, the analytical model has been tested extensively using real world data, i.e., observed data on operational satellites. This testing has led to several refinements of the theory for optimum application in a real world environment.

In what follows we will present the refined analytical theory and discuss the appropriate interfaces with real world inputs in order to optimize the performance in this environment. Results of predictions of the analytical theory versus real world satellite trajectories are given. As a point of comparison, the same predictions are carried out with a numerical integration model.

2. Equations of Motion

We will take \( J_2 \) to be a first-order quantity while \( J_3, J_4 \), and drag will be taken as second order. The corresponding perturbing gravitational potential function is

\[
V = \frac{\mu}{r} \left[ J_2 \left( \frac{R}{r} \right)^2 \left( \frac{3}{2} \sin^2 \phi - 1/2 \right) + J_3 \left( \frac{R}{r} \right)^3 \left( \frac{5}{2} \sin^3 \phi - \frac{3}{2} \sin \phi \right) + J_4 \left( \frac{R}{r} \right)^4 \left( \frac{5}{8} \sin^4 \phi - \frac{30}{8} \sin^2 \phi + \frac{5}{8} \right) \right],
\]

where \( \mu \) is the product of the gravitational constant and the mass of the Earth, \( r \) is