ON GOUĐÀS’ SURFACES IN THE MAGNETIC-BINARY PROBLEM

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Abstract. The containment property Goudas’ surfaces has been studied in this paper. Physical implications of this property are consequently discussed. A general relationship is obtained, which connects the sizes of those surfaces, the magnetic moments of two stars and their mean motion.

1. Introduction

The problem of the motion of the charged particles, within the magnetic field caused by a magnetic-binary star, has been deeply studied in the recent past (Mavraganis, 1977; Goudas, Leftaki, Petsagourakis, 1985).

Unavoidable simplification of the problem is to consider the model composed by two magnetic dipoles performing a circular uniform motion about a fixed point O resting anywhere along the straight-line connecting the dipoles. If the dipoles are associated with two stars $S_1$ and $S_2$, then their centre of mass is the above mentioned fixed point $O$.

As regards the charged particles, owing the weakness of newtonian fields in comparison to the Lorentz forces, only these are considered.

The assumed frame of reference $O, x, y, z$, to describe the motion of a charged particle $P$, having mass $m$ and charge $q$, is rotating in the same direction and with the same angular velocity $\omega$ as the stars $S_1$ and $S_2$, which in this frame are taken to stay at rest on the $Ox$-axis (Figure 1). The masses of the stars are $m_1$ and $m_2$, having $M = m_1 + m_2$, and the mutual fixed distance is $D$. Owing to the third Kepler law we shall have:

$$\omega^2 D^3 = GM,$$  \hspace{1cm} (1)

here $G$ is the universal gravitation constant. The distance between $S_1$ and $O$ will be $a = (m_2/M)D$, while the distance, between $S_2$ and $O$ will be $b = m_1/M$. If we adopt $D$ as the unity of length, we will have $a = \mu = m_2/m_1 + m_2$ and $b = 1 - \mu = m_1/m_1 + m_2$. With this choice the (1) becomes $\omega^2 = GM$.

At the centre of two stars is located a simple magnetic dipole, having the axis parallel to $Oz$-axis. The two magnetic moments $m_1$ and $m_2$ will have the components: 0, 0, $a$ and 0, 0, $\lambda a$ in which $a > 0$ and $\lambda$ any real number (Goudas, Petsagourakis, 1984).

As a consequence of the study of the charged particles motion, C. L. Goudas has shown that spaces of trapping exist in which the charged particles perform three-dimensional motion. These containment regions are often with heart-shaped forms,
one behind each dipole, and furthermore they are limited by surfaces which will be denoted, in this article, as Goudas' surfaces.

The aim of this investigation is to study these surfaces, from the point of view of the containment action, to obtain an analytical global relationship among the angular velocity, the average value of the magnetic induction (over the volume of the before-said region) and finally the size of the same region.

2. Analysis of the Problem

Let us consider the particle motion within the containment region; at the instant \( t \) it arrives at the surface and then it is reflected back. In the instant of impact \( t \) we will have \( \dot{x} = \dot{y} = \dot{z} = 0 \), and the fundamental motion equations will be:

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\begin{align*}
\dot{x} &= \omega^2 x - \varepsilon \omega \frac{\partial A_x}{\partial x} y + \varepsilon \omega A_y + \varepsilon \omega \frac{\partial A_y}{\partial x} x, \\
\dot{y} &= \omega^2 y + \varepsilon \omega \frac{\partial A_y}{\partial y} x - \varepsilon \omega A_x - \varepsilon \omega \frac{\partial A_x}{\partial y} y, \\
\dot{z} &= -\varepsilon \omega \frac{\partial A_x}{\partial z} y + \varepsilon \omega \frac{\partial A_y}{\partial z} x.
\end{align*}
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