LIBRATION EFFECTS FOR RETROGRADE SATELLITES IN THE RESTRICTED THREE-BODY PROBLEM

I: Circular Plane Hill's Case

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Abstract. Numerical explorations of the restricted problem have shown that for stable large non-periodic retrograde satellite orbits, the motion can be decomposed into a fast 'reference motion' and a slow libration around $B_2$. We study here this libration in the circular plane Hill's case, for which the 'reference motion' is elliptic. We establish the equations of motion for the coordinates of the centre of this ellipse. We find two integrals of motion: the first is the semi-major axis of the ellipse; the second is essentially Jacobi's integral, translated into the new coordinates. We give a formula for the period of the libration and we find its limiting value for small libration amplitudes. A numerical verification gives very good agreement for all these results.

1. Introduction

In his numerical exploration of the restricted three-body problem, Hénon (1970) has shown that, in the circular plane Hill's case, stable large non-periodic orbits associated to the simple-periodic family $f$ (i.e. retrograde satellites of $B_2$) could be approximately described as an elliptic retrograde fast 'reference motion', with an axis ratio 2:1 and a period of the order of $2\Pi$, whose centre slowly librates around $B_2$ on a very elongated vertical oval (see Figure 1). We have shown that, for the Sun–Jupiter plane case (Benest, 1971) and for the circular plane case with different values of the mass ratio $\mu$ – at least up to $\mu=0.054$ – (Benest, 1974), this phenomenon of libration still exists, although the 'reference motion' and the trajectory of its centre cannot be described any more by simple ellipses (see Figure 2).

Our aim is to study the theoretical reasons of this libration; here we shall consider only the simplest case, i.e. the circular plane Hill's problem.

2. Equations

We call $B_1$ and $B_2$ the two massive bodies (with $m_1 \geq m_2$) and $B_3$ the third body such as $m_3$ is negligible with regard to $m_1$ and $m_2$.

In rotating axes with the origin in $B_2$, the equations of the circular plane Hill's case are:

\[
\begin{align*}
\dot{x} &= u, \\
\dot{y} &= v, \\
\dot{u} &= 2v + 3x - x/q^3, \\
\dot{v} &= -2u - y/q^3,
\end{align*}
\]
Fig. 1. An example of libration in Hill's case: $X_0 = 15$, $V_0 = -30.1$ (Hill's coordinates) (rotating axes, origin in $B_2$).