MATRIX FORMULATION OF THE PICARD METHOD FOR PARALLEL COMPUTATION

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(Received 12 August, 1980; accepted 29 April, 1982)

Abstract. The increasing availability of computing machines capable of parallel computation has accelerated interest in numerical methods that exhibit natural parallel structures. In particular, the parallel structure of the Picard method of successive approximations for the numerical solution of ordinary differential equations allows straightforward adaptation of the method for use on parallel computers. A matrix formulation of the Picard method for parallel computation is presented here in which the numerical solution is obtained in truncated Chebyshev series. The application of the formulation to parallel processing computing machines is discussed.

1. Introduction

The increasing availability of reliable and efficient parallel and vector computing machines has begun to alter significantly the development of numerical methods. Many well-known and widely used algorithms can be restructured to take advantage of the parallel architecture of these computers. We describe how the Picard method of successive approximations can be restructured so that it is suitable for parallel computation. Moreover, the solution is obtained in the form of a finite or truncated Chebyshev polynomial series. Such a form of the solution is especially convenient for interpolation and for error estimation and control.

Many commonly used numerical algorithms for solving ordinary differential equations are serial or sequential in nature, particularly those used for the initial value problem. Typically, these algorithms advance the solution in a stepwise fashion and employ evaluation of the derivatives during each step. Several techniques are suggested in the literature for introducing parallelism for such algorithms. Nievergelt (1964) suggests that a given range of the independent variable be segmented into several smaller intervals and that numerical solutions be generated using conventional methods over each interval independently (and concurrently). This technique requires that approximate initial conditions be guessed at the beginning of each small interval (except the first) before the parallel integration commences. After the integration has been accomplished, these initial conditions are adjusted by interpolation so that the numerical solution is continuous over the given range. This technique is
similar to 'parallel shooting' techniques for solving boundary value problems (Keller, 1968).

Miranker and Lininger (1967) achieve a high degree of parallelism for classical predictor-corrector methods and for explicit Runge-Kutta methods by rearranging the order of the evaluations of the derivatives so that certain operations can be performed in parallel. Worland (1976) describes a class of block implicit methods which achieve an even greater degree of parallelism in much the same way. Franklin (1978) introduces an 'equation segmentation method' that allows a system of differential equations to be integrated efficiently in parallel provided that the system is loosely coupled and that each equation requires approximately the same amount of computation.

An implicit method for the numerical solution of initial and boundary value problems using Picard iteration with Chebyshev series was originally developed and implemented by Clenshaw and Norton (1963) using conventional serial computing algorithms. Extensions, modifications, and applications of the method have been introduced by Nacozy and Feagin (1971, 1972), Feagin (1972), and Nacozy (1972), among others. The method presented here for the solution of initial value problems using parallel computation employs a sequence of matrix operations originally formulated by Feagin (1972) as an outgrowth of work by the authors. The extension of the method presented here to the parallel solution of boundary value problems is straightforward (see, for example, Feagin, 1972; Nacozy and Feagin, 1972).

2. The Picard Method with Chebyshev Series

We first review the Picard method of successive approximations using Chebyshev polynomial series. Consider the problem of solving a non-linear, first-order system of ordinary differential equations,

\[ y'(\tau) = g[y(\tau), \tau] \]  \hspace{1cm} (1a)

where \( y \) and \( g \) are vectors with \( m \) components,

\[ y = \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^m \end{bmatrix} \quad \text{and} \quad g = \begin{bmatrix} g^1 \\ g^2 \\ \vdots \\ g^m \end{bmatrix} \]

\( \tau \) the independent variable, and where the prime indicates differentiation with respect to \( \tau \). The variable \( \tau \) is defined in the uniform interval \(-1 \leq \tau \leq 1\). The system (1a) is subject to the initial condition,

\[ y(-1) = 0. \]  \hspace{1cm} (1b)

A more general form of the interval of interest can be defined by the linear transforma-