COMMENTS ABOUT THE DIRECT PERTURBATIONS OF VENUS AND MARS ON THE MOON'S MOTION*

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Abstract. In a previous paper (Standaert, 1980) we have described an algorithm to compute the direct perturbation of the planets on the Moon's motion. A short summary of this algorithm is presented in Section 2 of this paper. Our first results permit us to present some complements and comments about these computations.

The algorithm is based upon the Lie transform method and is implemented using Chapront's ELP as solution of the main problem with the partial derivatives of Henrard's Semi-Analytical Lunar Ephemeris (SALE), and Bretagnon's mean Keplerian orbit.

An analysis of truncation errors in intermediate results is presented including the resonance effects. The final accuracy of the solution is intended to be about 0.0005 for terms of period up to 2000 yr in the case of Venus and up to 5000 yr in the case of Mars.

The effects of second-order terms in the masses are investigated. Only those depending upon the second derivatives of the mean motions are found to be significant to the given accuracy and are included.

1. Introduction

In the last 10 years, the development of a new analytical or semi-analytical theory of the lunar motion has been one of the activities of many research centers. The reason is that Brown's theory, which has been in use with some corrections and additions, in the computation of the Ephemerides, is no longer sufficient for the reduction of the very accurate data now available.

Although it is believed that the major weaknesses of Brown's theory lie in the planetary contributions, most of these new theories are concerned with the main problem (Deprit et al., 1971; Chapront-Touzé, 1974, 1979; Henrard, 1979).

Recent comparisons (Chapront-Touzé and Henrard, 1980) prove that the best nominal solution for the main problem is actually the ELP solution elaborated by J. and M. Chapront at the 'Bureau des Longitudes' (Chapront-Touzé, 1974, 1980), and that the best derivatives are those obtained by J. Hernard at the FUNDP of Namur (SALE Theory) (Henrard, 1978, 1979). Therefore, we used the more accurate result of these two theories. The computations of the planetary perturbations need the knowledge of the mean Keplerian orbits of the planets. We have based our work on the results obtained by P. Bretagnon and J. L. Simon at the 'Bureau des Longitudes' (Simon and Bretagnon, 1975).

In a previous paper (Standaert, 1980), we described the method we use to compute the planetary corrections taking care of frequencies problems. This method will be summarized in the following section. The aim of this paper is to give some indications.

about the estimated accuracy of our results and to examine the different contributions into the second order of the mass ratio. We have computed the direct action of Venus and Mars to an accuracy of about 0'0005 for terms of period up to 2000 yr in the case of Venus and up to 5000 yr in the case of Mars. Our results have been compared with those obtained independently by J. and M. Chapront (Chapront and Chapront-Touzé, 1980).

2. Formulation of the Problem

We may describe the Hamiltonian of the direct perturbations of a planet in the following form (Standaert, 1980):

\[ \mathcal{H} = H_{MP} + P_1, \]

where

\[ H_{MP} = \frac{m_0 m_1}{m_0 + m_1} H_1 + k^2 m_2 \left( m_0 \left( \frac{1}{r_2} - \frac{1}{r_{02}} \right) + m_1 \left( \frac{1}{r_2} - \frac{1}{r_{12}} \right) \right) \]

is the Hamiltonian of the main problem and where

\[ P_1 = k^2 m_3 \left( m_0 \left( \frac{1}{r_3} - \frac{1}{r_{03}} \right) + m_1 \left( \frac{1}{r_3} - \frac{1}{r_{13}} \right) \right) \]

is the direct planetary perturbation.

Fig. 1. Geometry of the perturbation

In a first step, we shall restrict our study to the first order. Considering the Hamiltonian (1) as function of the modified Delaunay's elements \( (\lambda, p, q; L, P, Q) \) of the orbit of the Moon around the Earth, and eliminating the dependency on the angles of the main problem with the help of a Lie transform, we have the following formulation

\[ \tilde{H}_{MP} = \sum_{i=1}^{3} n_i y_i + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} n_{ij} y_i y_j, \]