Influence of cross phase modulation and four-photon parametric mixing on the relative motion of optical pulses

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Received 18 May; accepted 24 August 1991

Two mechanisms of self-confinement of wave packets on different carrying frequencies in monomode optical fibre are investigated. It is shown that the equations describing the processes possess a Hamiltonian structure. A new class of asymptotically free soliton solutions are found. Two spectral bandwidths for self-confinement of the optical pulses are determined.

1. Introduction
Investigating the nonlinear interactions of picosecond optical pulses with different carrying frequencies in monomode optical fibre, both linear effects (dispersion and the group delay between the waves) must be taken into consideration. Also some additional nonlinear effects, such as cross phase modulation (CPM), four-photon parametric mixing (FPM), appear. As a rule, for optical pulses with soliton characteristics the group delay dominates among all other effects. Nevertheless, there are some important cases when it is possible to compensate for the group delay through the nonlinear mechanisms. The effects of self-confinement between picosecond optical pulses due to CPM in multimode [1, 2] and birefringent [3-11] optical fibres have been discussed in many papers, using \( N \) coupled nonlinear Schrödinger equations (CNLS). Some authors [6, 7, 11] try to reduce this infinite-dimensional Hamiltonian system to a finite-dimensional one, as functions of only pulse positions and widths and their derivatives with respect to distance for soliton-like pulses (particle models). In this way it is practically impossible to introduce an exact potential of interaction. This potential depends on higher-order moments [11]. In our view it is more convenient to work with the infinite-dimensional Hamiltonian formalism and dependence on the moments as functionals of the localized square-integrated functions. In this paper there is an exact expression of this potential [12, 13], and we investigate the influence of both CPM and FPM on the relative movement of the optical pulses.

2. Basic equations
We assume that the carrying frequencies and wavenumbers of three wave packets satisfy

\[
2\omega_3 = \omega_1 + \omega_2 \quad \text{and} \quad 2K_3 - K_1 - K_2 = \Delta K
\]
For the slowly varying amplitudes of electric field we obtain the coupled equations [14]

\[
\begin{align*}
\frac{i}{2} \left( \frac{i}{u_1} + \frac{1}{u_1} \frac{\partial A_1}{\partial t} \right) &= \frac{1}{2} k_1^2 \frac{\partial^2 A_1}{\partial t^2} + \frac{n_2 k_1}{2n_0} |A_1|^2 A_1 + 2|A_2|^2 A_1 \\
&+ 2|A_3|^2 A_1 + A_1^* A_2^* \exp (i\Delta Kz) \\
\frac{i}{2} \left( \frac{i}{u_2} + \frac{1}{u_2} \frac{\partial A_2}{\partial t} \right) &= \frac{1}{2} k_2^2 \frac{\partial^2 A_2}{\partial t^2} + \frac{n_2 k_2}{2n_0} |A_2|^2 A_2 + 2|A_1|^2 A_2 \\
&+ 2|A_3|^2 A_2 + A_2^* A_1^* \exp (i\Delta Kz) \\
\frac{i}{2} \left( \frac{i}{u_3} + \frac{1}{u_3} \frac{\partial A_3}{\partial t} \right) &= \frac{1}{2} k_3^2 \frac{\partial^2 A_3}{\partial t^2} + \frac{n_2 k_3}{2n_0} |A_3|^2 A_3 + 2|A_1|^2 A_3 \\
&+ 2|A_2|^2 A_3 + 2A_1 A_2 A_3^* \exp (-i\Delta Kz)
\end{align*}
\]

where

\[
u_i = \left( \frac{\partial k}{\partial \omega} \right)^{-1}_{v_i} = c \left[ n - \lambda_i \left( \frac{\partial n}{\partial \lambda} \right) \right]^{-1}
\]
is the group velocity of the \(i\)th wave packets and

\[
k'' = \left( \frac{\partial^2 k}{\partial \lambda} \right)_{v_i} = \frac{\lambda_i^2}{2\pi c^2} \left( \frac{\partial^2 n}{\partial \lambda^2} \right)_{v_i}
\]

the dispersion, \(n_2\)-Kerr coefficient. In a dimensionless coordinate system relative to one of the waves (pump \(A_3\)), the system of Equation 2 are transformed to the form

\[
\begin{align*}
\frac{i}{2} \left( \frac{i}{v_i} + \frac{1}{v_i} \frac{\partial A_i}{\partial t} \right) &= \frac{1}{2} D_i \frac{\partial^2 A_i}{\partial t^2} + R_i |A_i|^2 A_i + 2|A_2|^2 A_i \\
&+ 2|A_3|^2 A_i + A_i^* A_2^* \exp (i\Delta Kz) \\
\frac{i}{2} \left( \frac{i}{v_2} + \frac{1}{v_2} \frac{\partial A_2}{\partial t} \right) &= \frac{1}{2} D_2 \frac{\partial^2 A_2}{\partial t^2} + R_2 |A_2|^2 A_2 + 2|A_1|^2 A_2 \\
&+ 2|A_3|^2 A_2 + A_2^* A_1^* \exp (i\Delta Kz) \\
\frac{i}{2} \left( \frac{i}{v_3} + \frac{1}{v_3} \frac{\partial A_3}{\partial t} \right) &= \frac{1}{2} D_3 \frac{\partial^2 A_3}{\partial t^2} + R_3 |A_3|^2 A_3 + 2|A_1|^2 A_3 \\
&+ 2|A_2|^2 A_3 + 2A_1 A_2 A_3^* \exp (-i\Delta Kz)
\end{align*}
\]

where \(1/v_i = [(1/u_i) - (1/u_3)]/\tau \), \(i = 1, 2\), is the group delay of the \(i\)th wave relative to the pump \(A_3\), \(z = z; t = (t - z/v_i)/\tau; A_i = A_i/A_3 \) and \(R_i = z_d/z_{3i}^n\),

\[
z_d = \tau_0^2/k_3^2 \quad z_{3i}^n = n_2 k_3 |A_i|^2/2n_i \quad D_i = k''_i/k_3^2
\]

3. Influence of CPM
When the conditions of Equations 1 are not satisfied, Equations 4 are reduced to the CNLS

\[
\begin{align*}
i \left( \frac{i}{v_i} + \frac{1}{v_i} \frac{\partial A_i}{\partial t} \right) &= \frac{1}{2} D_i \frac{\partial^2 A_i}{\partial t^2} + R_i |A_i|^2 A_i + 2 \sum_{j=1}^{n} A_j |A_i|^2 A_i \\
&
\end{align*}
\]

where \(i = 1, \ldots, n, n = 3, 1/v_3 = 0\) and \(D_3 = 1\).