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# Mathematical model of honeycomb construction

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**Abstract.** We developed a mathematical model and an algorithm for numerical treatment of a model of honeycomb construction in a beehive. The model contains essential features of the bee-bee and bee-wax interactions, and in a qualitative way captures the dynamics of parallel comb construction. The construction is represented by a set of dynamical coupled partial differential equations for the density of bees situated on the hive ceiling, and the quantity of wax distributed by the bees. A spectral algorithm is invented for treatment of these equations, based on a modified thin-sheet gain scheme and a fast Fourier transform technique.

**Key words:** Comb building — Self-organization — Gain-sheet fast Fourier procedure

## 1. Introduction

The construction behavior of social insects is a very complex and highly cooperative phenomenon. It is well known that *Apis mellifica* has a strong tendency to construct parallel combs in a hive. Their construction technique have been extensively studied, and the corresponding literature is abundant [1-3]. However, most of the accounts are observational and experimental in nature; what seems to be lacking is a mathematical description or a model which would in a simple but adequate manner display the essential dynamical features of the construction. In this paper we propose such a model and present a detailed account of the numerical procedure needed for treatment of the model. The details of the model, together with a description of the relevant bee-bee and bee-wax interactions, as well as a simple explanation of the nature of biological and dynamical mechanisms in the construction, will be published elsewhere [4]. Our modeling follows Darchen's [2] observations and conclusions based on his experimental work.

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In a phenomenological approach, the model is represented by a set of nonlinear partial differential equations for the density of bees parallel to the $x0z$ plane (the coordinate system is placed so that $x$ and $y$ axes lie in the ceiling plane, with the $z$-axis pointing downwards), the density of bees parallel to the $y0z$ plane, and the distribution of wax on the ceiling. The numerical method invented for solution of the equations presents a modification of the thin gain-sheet fast Fourier procedure developed originally for optical mode calculations [5-7]. In its nature the method represents a spectral algorithm geared for treatment of multicomponent multidimensional nonlinear spatiotemporal physical or biological or chemical systems. It contains enough novel features to warrant presentation to the computer oriented scientists in the field.

This paper is organized in the following way. In Sect. 2 the model is introduced. Section 3 deals with a description of the numerical method applied to the model. The last section contains some of our results and conclusions.

## 2. The model

In an empty beehive a swarm of bees, hanging from the ceiling, constructs more or less parallel and equidistant combs (Figs. 1, 2). The construction is a social phenomenon — a small group of honeybees does not construct. Such a task requires a sufficient number of bees (about 100 in presence of a queen, and 10 000

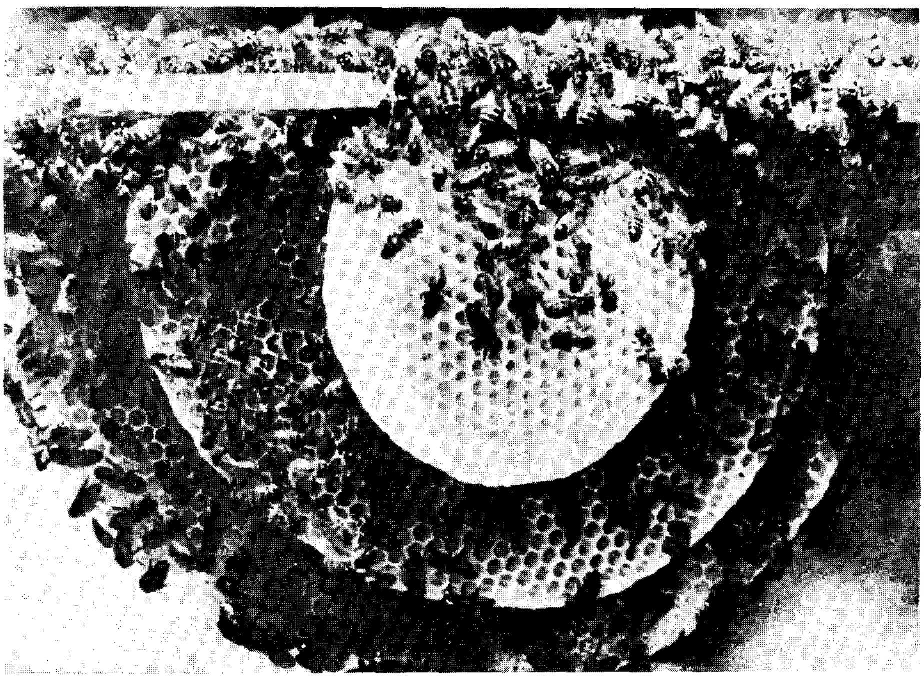

**Fig. 1.** The actual building of parallel combs by a swarm of *Apis mellifica* (sideview). Three comb ribs may be distinguished on the photo: the central comb is the largest visible disc, and the lateral combs are the two smaller disci (*after M. Lindauer 1961* [8], reprinted by permission)