BOUNDED MOTION IN A TWO-BODY SYSTEM CONSISTING OF A SOLID BODY AND A MATERIAL POINT

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Abstract: In this paper we prove the existence of bounded motions for an isolated system consisting of a solid body \( B_1 \) and a material point \( B_2 \) moving under their mutual gravitational attraction. We also consider the special case where the mass of \( B_1 \) is symmetrically distributed with respect to three mutually perpendicular planes passing through its mass center and \( B_2 \) moves on one of these planes. We study the types of the regions of possible motion and the ways of their evolution as the energy or the angular momentum of the system changes. As an example we present some results from a numerical study of the case where \( B_1 \) is a homogeneous prolate spheroid.

1. Introduction

It is known that in the gravitational problem of two point bodies bounded motion exists and is of ring type. The study of the existence and types of bounded motion in the general case where both bodies are arbitrary solids is of obvious physical interest. In this paper we find the necessary and sufficient condition for the possible configurations of two solids for given energy and angular momentum of the system. We apply this condition to the case where one of the bodies \( (B_2) \) is a point body and we prove the existence of bounded motions. To simplify the study of the types and evolution of the regions of possible motion we consider the case where the mass of \( B_1 \) is symmetrically distributed with respect to three mutually perpendicular planes passing through its mass center. To support the qualitative results of this study we present some results from a numerical study of the case where \( B_1 \) is a homogeneous prolate spheroid. We show that in this case the regions of possible motion present a great variety of forms. This work can be considered as an extension of the work of Michalodimitrakis and Bozis (1985) where \( B_1 \) was modelled as a homogeneous cube. As far as we know, there are no other studies of the regions of possible motion in the general gravitational problem of two bodies. Klat and Marchal (1978) studied the existence of planar motions of the mass center of two solids.

2. The Possible Configurations of a System of Two Arbitrary Solids for Given Energy and Angular Momentum

We consider an isolated system of two arbitrary solid bodies \( B_1 \) and \( B_2 \). Let 0 be the mass center of the system and \( 0X, 0Y, 0Z \), the main axes of inertia corresponding to a given configuration of \( B_1, B_2 \) at time \( t \).

The energy integral of the system is

\[ T + V = \text{const.} \equiv E, \]

where the gravitational potential energy $V$ is a function of the configuration only and $T$ is the kinetic energy.

The angular momentum integral is

$$\int dm(\tilde{r} \times \tilde{v}) = \text{const.} \equiv \mathcal{L},$$

where $\tilde{r}(x, y, z)$ and $\tilde{v}$ are, respectively, the position vector and velocity of the mass element $dm$.

Let $\omega$ be the rotation vector that gives to the system an angular momentum equal to $\mathcal{L}$, i.e.

$$\mathcal{L} = \int dm(\tilde{r} \times \tilde{v}) = \int \tilde{r} \times (\omega \times \tilde{r}).$$

Then $T$ can be written in the form

$$T = E - V = \frac{1}{2} \int dm v^2 = \int dm(\omega \times \tilde{r})^2 + \int dm(\tilde{v} - \omega \times \tilde{r})^2. \quad (1)$$

The first integral of the second member of (1) is equal to the (rotational) kinetic energy of the system if, at the time $t$, $B_1$ and $B_2$ were rigidly connected. Consequently we have

$$T_{\text{rot}} = \frac{1}{2} \left( \frac{L_x^2}{A} + \frac{L_y^2}{B} + \frac{L_z^2}{C} \right),$$

where $A, B, C$ (with $A \geq B \geq C$) are the main moments of inertia of the system and the energy integral takes the form

$$T' + V' = E,$$

where

$$T' = \int dm(\tilde{v} - \omega \times \tilde{r})^2 \geq 0,$$

and

$$V' = V + \frac{1}{2} \left( \frac{L_x^2}{A} + \frac{L_y^2}{B} + \frac{L_z^2}{C} \right),$$

is the effective potential energy.

Since $T' \geq 0$ we conclude from the above that

$$E - V \geq \frac{1}{2} \left( \frac{L_x^2}{A} + \frac{L_y^2}{B} + \frac{L_z^2}{C} \right). \quad (2)$$

It is easy to verify that the configuration of interest is, for suitable velocities, compatible with $E$ and $\mathcal{L}$ if and only if (2) is satisfied.

Since $A \geq B \geq C$, if we are not interested in the relative orientation of the