ON THE MINIMUM DISTANCE BETWEEN TWO KEPLERIAN ORBITS WITH A COMMON FOCUS

P. A. DYBCZYŃSKI, T. J. JOPEK, and R. A. SERAFIN

Astronomical Observatory of the A. Mickiewicz University Słoneczna 36, 60–286 Poznań, Poland

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Abstract. The methods used so far for determination of the closest approach between two orbits are discussed, and corrected versions of two of them are presented.

1. Introduction

The problem of the minimum distance between two orbits may be encountered for example in:

- determination of close approaches of comets and planetoids to the Solar System planets,
- calculation of the cometary radiants,
- investigation of the evolution of meteor streams.

The problem of the minimum distance has been considered by several authors, for example by Dubyago (1949), Kramer (1953), Lazović (1967, 1981), Sitarski (1968), Babadjanov et al. (1980), Murray et al. (1980), and Hoots et al. (1984). However, methods described by these authors have some disadvantages from the viewpoint of practical computer calculations. These disadvantages result from the necessity of having suitable initial values (Lazović (1967, 1981), Murray et al. (1980), Hoots et al. (1984)) or from the limited applicability of the methods proposed (Dubyago (1949), Sitarski (1968), Babadjanov et al. (1980)).

In this paper we propose a solution of the problem of determination of the minimum distance between two arbitrary common focus orbits which does not have these disadvantages.

2. Preliminaries

Let \( \Sigma_1 \) and \( \Sigma_2 \) be arbitrary common focus Keplerian orbits (ellipses, parabolas or hyperbolas) defined by the perihelion distance \( q_k \), the eccentricity \( e_k \), the longitude of the ascending node \( \Omega_k \), the argument of the perihelion \( \omega_k \) and the inclination \( i_k \), for \( k = 1, 2 \), respectively.

The problem of determination of the closest approach between orbits \( \Sigma_1 \) and \( \Sigma_2 \) is

reduced to minimization of the function:

\[ D(f_1, f_2) = (\tilde{r}_2 - \hat{S}\tilde{r}_1)^T(\tilde{r}_2 - \hat{S}\tilde{r}_1), \]  

where \( f_1, f_2 \) are the true anomalies,

\[ \tilde{r}_k = r_k(\cos f_k, \sin f_k, 0)^T, \]

\[ r_k = \frac{q_k(1 + e_k)}{1 + e_k \cos f_k}, \]

\[ k = 1, 2, \]

and \( \hat{S} = \{s_{ij}\}, i, j = 1, 2, 3 \) is the orthogonal matrix of the transformation between the \( \Sigma_1 \) orbital coordinate system and the \( \Sigma_2 \) one. Elements of \( \hat{S} \) are functions of \( \Omega_k, \omega_k, i_k \) \((k = 1, 2)\) only.

From definition (1) it follows that:

\[ \forall (f_1, f_2) \in U \quad D(f_1, f_2) \geq 0, \]

thus there obviously exists such a value \( D^* \) that:

\[ D^* = \inf\{D(f_1, f_2) | (f_1, f_2) \in U\}, \]

where \( U \) is the domain of the function \( D \).

Every pair \((f_1^*, f_2^*) \in U\) satisfying the equation:

\[ D(f_1^*, f_2^*) = D^*, \]

are solutions to our problem. The set of all such pairs we denote by \( U^* \). The existence of at least one pair \((f_1^*, f_2^*) \in U^*\) is ensured when the domain \( U \) and the function \( D \) satisfy assumptions of the Weierstrass theorem. Namely, if \( U \) is a compact set and \( D \) is a continuous function over \( U \) then \( D \) takes a minimum value somewhere in \( U \).

3. Discussion of the Previously Used Methods

In all the papers quoted in the Introduction, the necessary and sufficient conditions of the local minimum existence were used, i.e.:

\[ \frac{\partial D}{\partial f_1} = 0, \quad \frac{\partial D}{\partial f_2} = 0, \quad (\text{5.1}) \]

\[ \frac{\partial^2 D}{\partial f_1^2} > 0 \quad \frac{\partial^2 D}{\partial f_2^2} > 0, \quad \left(\frac{\partial^2 D}{\partial f_1 \partial f_2}\right) > 0. \quad (\text{5.2}) \]

\[ \frac{\partial^2 D}{\partial f_1^2} \frac{\partial^2 D}{\partial f_2^2} - \left(\frac{\partial^2 D}{\partial f_1 \partial f_2}\right)^2 > 0. \quad (\text{5.3}) \]