A PRODUCTION FUNCTION FOR AUSTRIA
EMPHASIZING ENERGY

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1 COBB-DOUGLAS FUNCTION

The Cobb-Douglas function recommends itself as a possible macroeconomic production function through its simplicity. It is usually defined as

\[ X = AK^{a_1}L^{a_2} = F(K, L) \]  

(1.1)

with \( X \) as index of output (gross national product), \( K \) an index of capital (investments minus depreciations) and \( L \) an index of labour (number of employees). The constants \( A, a_1 \) and \( a_2 \) are to be estimated. The parameter \( \alpha \),

\[ \alpha = a_1 + a_2 \]  

(1.2)

indicates for \( \alpha < 1 \) decreasing, for \( \alpha = 1 \) constant, for \( \alpha > 1 \) increasing returns of scale. Because of homogeneity, we obtain

\[ \alpha X = K \frac{\partial F}{\partial K} + L \frac{\partial F}{\partial L} \]  

(1.3)

\( \alpha_1 \) and \( \alpha_2 \) are elasticities of production with respect to capital and labour:

\[ \alpha_1 = EX/EK \quad \text{and} \quad \alpha_2 = EX/EL. \]  

(1.4)

In the case of constant returns of scale, it is possible to reduce (1.1) to

\[ x = k^{\alpha_1} = f(k) \]  

(1.5)

with \( x = X/L \) (gross product per capita) and \( k = K/L \) (capital per capita).

When production is based on profit maximisation under perfect competition on both product and labour markets we obtain the well known result, that production

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is pushed to the point, where the marginal products are equal to factor prices $\rho$ for capital and $w$ for labour:

$$\frac{\partial F}{\partial K} = \rho \quad \text{and} \quad \frac{\partial F}{\partial L} = w.$$  \hfill (1.6)

Equation (1.3) reduces then to

$$X = \rho K + wL.$$  \hfill (1.7)

It gives the share between profit and wages of the gross national product, defined as consumption and investment in an economy without foreign trade and public activity. The production function combines the highly aggregated variables output, capital and labour in order to explain the macroeconomic tendencies. The difference between net and gross national product (without or with depreciation) therefore is of minor importance. Neutral technical change can be explained by the Cobb-Douglas function

$$X = A e^{uK^{a_1}L^{a_2}}$$  \hfill (1.8)

The simplicity of the Cobb-Douglas function, which is linear in the logarithms of the variables and involves only few parameters, recommends it for econometric investigations. In economic theory, it was already used by Wicksell (1893) and plays an important part in modern mathematical economics, especially in growth theory. The Cobb-Douglas function is a member of the more general class of CES production functions (constant elasticity of substitution), due to Arrow, Chenery, Minhas, Solow (1961). These functions are defined as

$$X^\lambda = a_1 K^\lambda + a_2 L^\lambda$$  \hfill (1.9)

The elasticity of substitution $\sigma$ is defined as

$$\sigma = \frac{\frac{d}{d\left(\frac{K}{L}\right)} \frac{K}{L}}{\frac{d}{d\left(\frac{\partial F/\partial L}{\partial F/\partial K}\right)} \frac{\partial F/\partial L}{\partial F/\partial K}}.$$  \hfill (1.10)

Under profit maximisation we get

$$\sigma = \frac{\frac{d}{d\left(\frac{K}{L}\right)} \frac{K}{L}}{\frac{d}{d\left(\frac{w}{\rho}\right)} \frac{w}{\rho}}.$$  \hfill (1.11)