An important problem in Location Theory is that of assigning plants to locations in an optimal manner. In the context of this problem, recognizing interplant transportation costs, Koopmans and Beckmann (1957) introduced the Quadratic Assignment Problem (QAP). It is shown in this paper that when the QAP is formulated as a cooperative location game, its core may be empty. By contrast, the core of the game corresponding to the linear assignment problem (where transportation costs are disregarded) is assured to be non-empty. Some conditions under which the core is non-empty are discussed.

1. Introduction

In a significant number of real world markets traded commodities come in large discrete units. Indivisibility is particularly prevalent in the case of production activities involving the assignment of resources to uses. The most compelling example of this kind is in the choice of locations for plants. It is consequently important to determine (both as a theoretical and practical matter) whether efficient resource allocation can occur in such cases.

In the typical assignment problem, resources are owned by several different agents and various uses have costs and benefits associated with them. An important question is whether this economic model has a non-empty core: is there an outcome which cannot be upset by the

* I am grateful to two anonymous referees of this journal for their comments.
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collusive action of any subset of agents acting by themselves? This
question has been answered in the affirmative for important classes
of models involving indivisibilities (see, e.g., Shapley and Shubik,
1972; Shapley and Scarf, 1974). In particular, the core of the game
corresponding to the linear assignment problem is always non-empty.
In exchange markets some authors have observed that the presence
of externalities can cause the core to be empty (e.g., Schotter, 1977;
Kaneko, 1983). In this note it is shown (for production economies)
that even the presence of a very innocuous form of interdependence
between agents—where scarce resources are needed for the transporta-
tion of intermediate commodities between plants—may cause the core
to become empty. There may be no way of assigning transportation
costs to agents such that the outcome cannot be upset by a subset
of agents. This is shown in section 3 below. First, in section 2, the
cooperative game corresponding to the Quadratic Assignment Problem
(QAP hereafter) is described.

2. The Location Game

Consider the following version of a location problem first intro-
duced by Koopmans and Beckmann (1957). There are \( m \) plants and
\( n \) locations. Let the set of plant owners be denoted by \( M \) and the
location owners (landlords) by \( N \). The location of a plant affects
its profitability so that different plant owners and landowners assign
(possibly different) values to each location. The \( p \)th plant owner values
location \( i \) at \( h_{ip} \) dollars and the \( i \)th landlord values it at \( c_i \) dollars.
The surplus from the match of \( p \) to \( i \) is \( h_{ip} - c_i \) (denoted \( a_{ip} \)) and
may be thought of as the value (or surplus) of the coalition \( \{i, p\} \) if
it is non-negative (the value being zero otherwise). The problem of
finding the assignment of plants to locations which maximizes the total
surplus is the well known linear assignment problem: find a permutation
matrix \( x \) which maximizes \( \sum a_{ip} x_{ip} \). Shapley and Shubik (1972)
showed that associated with the optimal assignment problem there is
always a payoff vector (an allocation of the total surplus among the
players) with the property that no group of players (in \( M \cup N \)) can,
by breaking off and acting on its own, obtain a total surplus which is
greater than what is given to it by this allocation. In other words, there
is always an allocation which belongs to the core of the location game.
This result uses the fact that if one assumes only that individual rows
and columns of \( x \) sum to one (i.e. we dispense with the requirement
that elements of \( x \) be either one or zero) then the linear programming
solution of the assignment problem will always be integral. The dual