An important problem in Location Theory is that of assigning plants to locations in an optimal manner. In the context of this problem, recognizing interplant transportation costs, Koopmans and Beckmann (1957) introduced the Quadratic Assignment Problem (QAP). It is shown in this paper that when the QAP is formulated as a cooperative location game, its core may be empty. By contrast, the core of the game corresponding to the linear assignment problem (where transportation costs are disregarded) is assured to be non-empty. Some conditions under which the core is non-empty are discussed.

1. Introduction

In a significant number of real world markets traded commodities come in large discrete units. Indivisibility is particularly prevalent in the case of production activities involving the assignment of resources to uses. The most compelling example of this kind is in the choice of locations for plants. It is consequently important to determine (both as a theoretical and practical matter) whether efficient resource allocation can occur in such cases.

In the typical assignment problem, resources are owned by several different agents and various uses have costs and benefits associated with them. An important question is whether this economic model has a non-empty core: is there an outcome which cannot be upset by the

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collusive action of any subset of agents acting by themselves? This question has been answered in the affirmative for important classes of models involving indivisibilities (see, e.g., Shapley and Shubik, 1972; Shapley and Scarf, 1974). In particular, the core of the game corresponding to the linear assignment problem is always non-empty. In exchange markets some authors have observed that the presence of externalities can cause the core to be empty (e.g., Schotter, 1977; Kaneko, 1983). In this note it is shown (for production economies) that even the presence of a very innocuous form of interdependence between agents—where scarce resources are needed for the transportation of intermediate commodities between plants—may cause the core to become empty. There may be no way of assigning transportation costs to agents such that the outcome cannot be upset by a subset of agents. This is shown in section 3 below. First, in section 2, the cooperative game corresponding to the Quadratic Assignment Problem (QAP hereafter) is described.

2. The Location Game

Consider the following version of a location problem first introduced by Koopmans and Beckmann (1957). There are m plants and n locations. Let the set of plant owners be denoted by M and the location owners (landlords) by N. The location of a plant affects its profitability so that different plant owners and landowners assign (possibly different) values to each location. The p\textsuperscript{th} plant owner values location i at $h_{ip}$ dollars and the i\textsuperscript{th} landlord values it at $c_i$ dollars. The surplus from the match of p to i is $h_{ip} - c_i$ (denoted $a_{ip}$) and may be thought of as the value (or surplus) of the coalition $\{i, p\}$ if it is non-negative (the value being zero otherwise). The problem of finding the assignment of plants to locations which maximizes the total surplus is the well known linear assignment problem: find a permutation matrix $x$ which maximizes $\sum_{i,p} a_{ip}x_{ip}$. Shapley and Shubik (1972) showed that associated with the optimal assignment problem there is always a payoff vector (an allocation of the total surplus among the players) with the property that no group of players (in M U N) can, by breaking off and acting on its own, obtain a total surplus which is greater than what is given to it by this allocation. In other words, there is always an allocation which belongs to the core of the location game. This result uses the fact that if one assumes only that individual rows and columns of $x$ sum to one (i.e. we dispense with the requirement that elements of $x$ be either one or zero) then the linear programming solution of the assignment problem will always be integral. The dual