Little Perfection and Complexity

ALEJANDRO NEME

Instituto de Matematica Aplicada, Universidad Nacional de San Luis, Ejercito de los Andes 950, 5700 San Luis, Argentina
Universitat Autònoma de Barcelona, Spain

Abstract: We consider the infinitely-repeated prisoners' dilemma with lexicographic complexity costs, where transitional complexity between states is included as one aspect of overall strategic complexity. We prove that a full folk theorem obtains in presence of any level of perfection of the equilibrium strategy, if the players consider off-equilibrium path payoff prior to minimizing complexity.

1 Introduction

Rubinstein [1986] and Abreu and Rubinstein [1989], modified the definitions of a game in order to incorporate complexity costs into the payoffs of the players. Complexity is measured by the number of machine states. In their models a substantial reduction in the number of outcomes supported by a Nash Equilibrium is obtained. For example, in the prisoners' dilemma game the set of equilibrium payoffs shrinks to two straight line segments.

Banks and Sundaram [1989] argued that this “complexity measure” does not allow for a sufficiently fine distinction between strategies. They provided an increasing complexity criterion, which captured the notion of transitional complexity. They showed that, in the Abreu-Rubinstein models, if players use any complexity measure which fulfills the “increasing complexity criterion”, then the Nash Equilibrium strategy is that, at each stage of the repeated game, the players adopt one-shot Nash Equilibrium actions. Their results imply, in the special case of the Prisoners' dilemma, that non-cooperation in all stages of the repeated game is the only Nash Equilibrium strategy.

When perfection is considered, the Banks-Sundaram results hold as long as complexity costs are more important than out-of-equilibrium payoffs. However, if a model with errors is used, payoffs near the equilibrium path are important and thus one should dealt with them before taking into account complexity costs.

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Kalai and Neme [1992] studied such a situation in the Abreu-Rubinstein models. They showed that, if the players are first concerned with maximizing their payoff and their payoff after deviations from the equilibrium path and then with minimizing their complexity cost, the full "Folk Theorem" is recovered.

The main contribution in this paper is to show that, once again, that it is the lack of perfection of Banks and Sundaram concept that brings about that discontinuity. We prove that, in the prisoners' dilemma game with the discounting payoff criterion, if the players are maximize their payoff and their payoff after deviations and then minimize the complexity costs of the strategy then the Full Folk Theorem is recovered. The result holds for any complexity measure that satisfies the Banks and Sundaram increasing complexity criterion. A weaker version of this result is proved for the average payoff criterion. We consider a measure of complexity defined by Banks and Sundaram as "measure 2" and prove that the Full Folk Theorem is recovered.

We do not consider all off-equilibrium path payoffs infinitely more important than complexity costs, but only the ones which are near the equilibrium path and are more likely to occur. We do not claim that a Full Folk Theorem is necessarily the real situation. But by pointing out the other side of the coin of Banks-Sundaram argument, by taking the opposite extreme to theirs. The truth must, in our opinion, lie some place in between. The question of how to deal with the trade off between complexity costs and perfection has not yet been answered in a satisfactory fashion.

2 The Model and a Discussion

Let \( G = (A_1, A_2, u_1, u_2) \) be a two-person game in normal form. Here \( A_i \) is a finite set of actions and \( u_i : A \to \mathbb{R} \) a utility function, with \( A = A_1 \times A_2 \). We will assume that the cardinality of \( A_i \), \( \#A_i \), is \( k \).

Let \( v_i \) be the player \( i \)'s minimax payoff. We define the following sets of payoff vectors:

\[
U = \{ u(a) : a \in A \}. \\
U^* = \text{Rational Convex Hull of } U. \\
V = \{ x \in U^* : x_i > v_i \}. 
\]

We describe a standard repeated game \( G^\infty(A, U), G^\infty,\alpha(A, U) \) associated with \( G \) as follows:

The set of histories of length 0 is a singleton set denoted by \( H^0 = \{ e \} \), its single element will be denoted by \( e \). Let \( H^m = A \times \ldots \times A \) be the set of histories of length \( m \)-times. \( H = \bigcup_{m=0}^\infty H^m \) is the set of all histories. For each player \( i = 1, 2 \); a strategy \( f_i \) is a function \( f_i : H \to A_i \). Let \( F_i \) be the set of all individual strategies of player \( i \), and \( F = F_1 \times F_2 \).