Abstract: This paper investigates a problem of the perfect equilibrium point in games in normal form by introducing a lexicographic domination of strategies for players, which turns out to be equivalent to a "local" domination of strategies. It is shown that a perfect equilibrium point is lexicographically undominated, and moreover that the lexicographic domination can narrow down the set of undominated equilibrium points in the ordinary sense when there are more than two players in a game.

1 Introduction

The purpose of this paper is to investigate a problem of the perfect equilibrium point in games in normal form by introducing a lexicographic domination of strategies for players.

Selten (1975) pointed out the undesirable property of a Nash equilibrium point that it may prescribe disequilibrium (i.e., non-payoff-maximizing) behavior for players on unreached information sets in a game in extensive form. In order to remove this difficulty, Selten introduced the concept of a perfect equilibrium point for a game in extensive form and also in normal form. Taking a point of view which regards complete rationality as a limiting case of incomplete rationality, Selten formulated a model of "slight mistakes" in which there exists a slight possibility for each player to deviate from his equilibrium strategy by mistake. Roughly speaking, a perfect equilibrium
point is defined to be a (Nash) equilibrium point in which each player's equilibrium strategy remains a best reply to all other players' strategies at every information set of game even if their deviations from the equilibrium point may occur with some slight probability. Selten showed that this definition of a perfect equilibrium point can accomplish his purpose above. Since the pioneering work of Selten, a perfect equilibrium point has been investigated by many authors as one of the most important concepts to explore rational behavior for players in noncooperative games (see Kreps and Wilson 1982, and van Damme 1983 etc.).

On the other hand, the concept of domination of strategies for players provides us with another useful approach for investigating rational behavior in games in normal form (see Luce and Raiffa 1957). A strategy $s$ for a player is said to dominate his another strategy $t$ if he would never obtain a strictly lower payoff from $s$ than from $t$ for all strategy combinations for the other players and could obtain a strictly higher payoff from $s$ than from $t$ for some strategy combination. Since it is hard to imagine that players employ their dominated strategies, an equilibrium point seems to be unreasonable if it involves dominated strategies for players. An equilibrium point is said to be undominated if it does not involve any dominated strategies for players.

In this paper, we will explore a relationship between the perfect equilibrium point and the domination of strategies. It is a well-known theorem that a perfect equilibrium point is undominated for $n$-person games in normal form, and that the converse is also true when $n = 2$. This theorem provides us with a useful characterization of a perfect equilibrium point. But, in the general case that $n \geq 3$, since the ordinary domination relation requires too much, the set of undominated equilibrium points may be so large that the theorem can not work well. It is often observed even that all equilibrium points of a game are undominated. Moreover, the ordinary domination is not so effective in eliminating disequilibrium behavior for players on unreached information sets in games in extensive form. Therefore we will introduce a weaker concept of domination between strategies, called a "lexicographic" domination, which turns out to be more appropriate for investigation of a perfect equilibrium point. We will show that the theorem above is still valid with respect to the lexicographic domination, and that the lexicographic domination can narrow down the set of undominated equilibrium points in the ordinary sense when there are more than two players in a game.

The lexicographic domination is defined to fit Selten's model of "slight mistakes" better than the ordinary one. Suppose that each player $i$ is confronted with the choice between two strategies $s_i$ and $t_i$, while all other players $j(\neq i)$ are expected to use a strategy combination $s = (s_1, ..., s_n)$ but they may deviate independently to other strategies $t_j$ with very small probability. In this situation, player $i$ must have concern about all possible simultaneous deviations by other players. However, since the simultaneous deviation by players in a group $D$ is more likely than that by players in a larger group $D'(\supset D)$, he must have more concern about the deviation by the smaller group in order to maximize his expected payoff. The lexicographic domination gives