Combined $B \rightarrow X_s \psi$ and $B \rightarrow X_s \eta_c$ decays as a test of factorization

Mohammad R. Ahmady$^a$, Emi Kou

Department of Physics, Ochanomizu University, 1-1 Otsuka 2, Bunkyo-ku, Tokyo 112, Japan (e-mail: ahmady@phys.ocha.ac.jp, kou@fs.cc.ocha.ac.jp)

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**Abstract.** We calculate the inclusive decays $B \rightarrow X_s \psi$ and $B \rightarrow X_s \eta_c$ using factorization assumption. To investigate the bound state effect of the decaying $B$ meson in these inclusive decays we take into account the motion of the $b$ quark using a Gaussian momentum distribution model. The resulting correction to free quark decay approximation is around 6% at most. Utilizing a potential model evaluation of the ratio of the decay constants $f_{X_s}/f_{\eta_c}$, it is shown that the ratio $R = \Gamma(B \rightarrow X_s \eta_c)/\Gamma(B \rightarrow X_s \psi)$ can be used as a possible test of factorization assumption.

Exclusive and inclusive nonleptonic $B$ decays to charmonium states are of special interest theoretically and experimentally. These decay channels, among other things, provide a powerful testing ground for color suppression and factorization in hadronic $B$ decays [1]. At the same time, exclusive modes of two body $B$ decay into $K$ meson resonances and charmonium states can provide an alternative examination of models for the treatment of hadronic form factors [2].

In this work, we focus on the inclusive two body decays $B \rightarrow X_s \psi$ and $B \rightarrow X_s \eta_c$, where $X_s$ is a final state hadron containing a strange quark. There is no experimental data on the latter decay at this time. However, as we point out, the eventual measurement of this inclusive decay channel can be used to test the validity of the factorization assumption in nonleptonic $B$ decays. In fact, one can show that the ratio $R = \Gamma(B \rightarrow X_s \eta_c)/\Gamma(B \rightarrow X_s \psi)$, calculated by using factorization, is independent of the QCD corrections and the scale ambiguity of the Wilson coefficients. In this context, $R$ depends only on the ratio of the decay constants $f_{\eta_c}/f_{\psi}$ for which we use an improved estimate obtained in a previous work [3].

The inclusive decays $B \rightarrow X_s \psi(\eta_c)$ are usually approximated with the free quark decays $b \rightarrow s \psi(\eta_c)$. To improve upon, in the present paper, we estimate the correction to this approximation by taking into account the motion of the $b$ quark inside the $B$ meson. For this purpose, we use a one parameter Gaussian momentum distribution for the $b$ quark which has previously been applied to inclusive semileptonic [4], rare dileptonic [5] and nonleptonic $B$ decays [6] (commonly known as ACCMM model in the literature). We present results for a range of the model parameter obtained from fits to experimental data.

Neglecting penguin operators, the relevant effective Hamiltonian for $B \rightarrow X_s \psi(\eta_c)$ can be written as:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb} \left[ C_1(\mu) \bar{b} \gamma^\mu (1 - \gamma_5) b \right] + H.C.,$$

(1)

where $i$ and $j$ are color indices and $C_1(\mu)$ and $C_2(\mu)$ are QCD improved Wilson coefficients. One then can use a Fierz transformation to write (1) in the following form:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb} \left[ \left( C_2(\mu) + \frac{3}{2} C_1(\mu) \right) \bar{b} \gamma^\mu (1 - \gamma_5) b + \bar{c} \gamma_\mu (1 - \gamma_5) c \right] + H.C.,$$

(2)

where $T^a (a = 1..8)$ are generators of $SU(3)_{\text{color}}$. We note that the first term in (2) is the product of two color singlet currents but the second term consists of two color octet currents. In the factorization assumption, only the color singlet quark current contributes to the production of colorless $c \bar{c}$ final state $\psi$ and $\eta_c$. Using the definition for the decay constant $(f_\psi)$ for the vector meson $\psi$:

$$f_\psi c^\mu = \langle 0 | c \gamma^\mu \psi | >,$$

(3)

($c^\mu$ is the polarization vector of $\psi$) the effective Hamiltonian for $B \rightarrow X_s \psi$ is obtained as follows:

$$H_{\text{eff}}^{B \rightarrow X_s \psi} = C f_\psi \bar{s} \gamma^\mu (1 - \gamma_5) b s_{\mu},$$

(4)
where
\[ C = \frac{G_F}{\sqrt{2}} V_{cb} V_{ts} \left( C_2(\mu) + \frac{1}{3} C_1(\mu) \right). \]  

(5)

Similarly, utilizing the definition for the decay constant \( f_{\eta_c} \) for the pseudoscalar meson \( \eta_c \):
\[ f_{\eta_c} g = \langle 0 | \bar{c} \gamma^\mu \gamma_5 s | \eta_c(q) \rangle, \]

(6)

results in the following effective Hamiltonian for \( B \rightarrow X_s \eta_c \) decay:
\[ H_{B \rightarrow X_s \eta_c} = C f_{\eta_c} g \gamma^\mu (1 - \gamma_5) b \bar{q}. \]

(7)

The Wilson coefficients \( C_1 \) and \( C_2 \) in (5) are calculated to the next-to-leading order in reference [7] resulting in
\[ a_2 = C_2(\mu) + 1/3 C_1(\mu) = 0.155 \]

for \( \mu = \overline{m}_B \approx 5 \) GeV. However, the branching ratio for the exclusive decay \( B \rightarrow K^* \) obtained from (4) requires \( a_2 \) to be roughly by a factor of two larger than the above value in order to agree with the experimental data [8]
\[ \text{BR}(B^+ \rightarrow K^+ \psi) = (0.101 \pm 0.014)\%. \]

This discrepancy between theoretical prediction and measurement could be due to two factors. On one hand, the \( \mu \)-dependence of the Wilson coefficients which arises from short distance QCD results in a significant uncertainty in the calculated decay rate in the context of factorization. In fact, phenomenologically, \( a_2 \) is treated as a free parameter to be determined from experiment [9]. On the other hand, one may question the validity of the factorization assumption which allows to infer (4) from the effective Hamiltonian (2). In other words, the second term in (2) which is nonfactorizable could have a significant contribution to the matrix element [10]. To disentangle these two factors and examine the factorization assumption, the ratio of the inclusive decays \( R \) can serve as a crucial testing ground. Aside from the cancellation of the Wilson coefficients in \( R \), this ratio is also free from the nonperturbative hadronic uncertainties which is usually associated with the theoretical calculations of exclusive decays [11].

Using (4) and (7), one can calculate the decay rates \( \Gamma(B \rightarrow s \psi(\eta_c)) \):
\[ \Gamma(b \rightarrow s \psi(\eta_c)) = \frac{C^2 f^2_{\eta_c}}{8 \pi m_b^2 m_{\psi}^2} g(m_b, m_s, m_{\psi}) \left[ m_b^2 (m_b^2 + m_{\psi}^2) - 2m_s^2 (2m_b^2 - m_{\psi}^2) + m_s^4 - 2m_b^2 m_{\psi}^2 \right], \]

(8)

\[ \Gamma(b \rightarrow s \eta_c) = \frac{C^2 f^2_{\eta_c}}{8 \pi m_b^2} g(m_b, m_s, m_{\eta_c}) \times \left[ (m_b^2 - m_s^2)^2 - m_{\eta_c}^2 (m_b^2 + m_s^2) \right], \]

(9)

where
\[ g(x, y, z) = \left[ 1 - \frac{y^2}{x^2} - \frac{z^2}{x^2} - 4 \frac{y^2 z^2}{x^4} \right]^{1/2}, \]

(10)

and \( m_b \) and \( m_s \) are \( b \) and \( s \) quark masses, respectively. The inclusive decay rates \( \Gamma(B \rightarrow X_s \psi(\eta_c)) \) are usually approximated by (8) and (9). However, in this work we estimate the bound state corrections to this approximation by taking into account the motion of the heavy \( b \) quark inside the \( B \) meson. We follow the ACCMM method [4] which incorporates the bound state effect in semileptonic \( B \) decays by assuming a virtual \( b \) quark inside \( B \) meson accompanied by an on-shell light quark. In the meson rest frame, the energy-momentum conservation leads to the following relation for \( b \) quark mass \( W \):
\[ W^2(p) = m_b^2 + m_q^2 - 2m_B \sqrt{p^2 + m_q^2}, \]

(11)

where \( m_q \) is the light quark mass and \( p \) is the 3-momentum of the \( b \) quark. Following [4], we also consider a Gaussian momentum distribution for the Fermi motion of \( b \) quark:
\[ \phi(p) = \frac{4}{\sqrt{\pi} p_F^2} e^{-p^2/p_F^2}. \]

(12)

The model parameter \( p_F \) determines the distribution width, and is related to the average momentum \( \langle p \rangle \).

At this point we would like to remark on the consistency of the above model with heavy quark expansion. Let us consider the average \( b \) quark mass \( \bar{m}_b \) defined as:
\[ \bar{m}_b = \int_0^{p_{\text{max}}} W(p) \phi(p) p^2 dp, \]

(13)

where \( p_{\text{max}} \) is the maximum kinematically allowed momentum. Using (13), one can derive an expansion of the \( B \) meson mass \( m_B \) in powers of \( \bar{m}_b \) as follows:
\[ m_B = \bar{m}_b + \frac{2p_F}{\sqrt{\pi}} + \frac{3p_F^2}{4m_b} + O \left( \frac{1}{\bar{m}_b^2} \right), \]

(14)

in which \( m_q = 0 \) is assumed. A comparison of (14) with the usual heavy quark expansion formula for heavy-light mesons, i.e.
\[ m_M = m_Q + \frac{A_1 + d_M A_2}{2m_Q} + O \left( \frac{1}{m_Q^2} \right), \]

(15)

reveals that once \( \bar{m}_b \) is identified with the mass of the heavy quark \( m_Q \) in (15), the nonperturbative parameters \( A \) and \( \lambda_1 \) of the heavy quark expansion and the model parameter \( p_F \) are connected as follows:
\[ A = \frac{2p_F}{\sqrt{\pi}}, \quad \lambda_1 = -\frac{3p_F^2}{2}. \]

(16)

The ACCMM model does not provide a corresponding term for the nonperturbative parameter \( \lambda_2 \) \( (d_M = 3, -1 \) for pseudoscalar and vector mesons, respectively) which is due to the spin interactions. This could be considered as a shortcoming of the model. However, the constraint imposed by (16), i.e. \( \lambda_1 = -3\pi/8A_2^2 \), is in reasonable agreement with quoted values for these parameters [12]. It is in this sense that we consider the above model to be consistent with heavy quark symmetries.

To incorporate the effects of the motion of the \( b \) quark in the inclusive decays \( B \rightarrow X_s \psi(\eta_c) \), we replace the \( b \)