CALCULATING THE SPATIAL AND ENERGY DISTRIBUTION OF THERMAL NEUTRONS IN A HETEROGENEOUS REACTOR

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A method is presented for determining the spatial and energy distribution of thermal neutrons within the cells of a heterogeneous reactor. If the effect of the asymmetric part of the scattering cross section on the change in neutron energy is neglected, the problem of determining the integral parameters of a cell may, in many cases, be reduced to the solution of monoenergetic Boltzmann equations for a certain set of energy values with a subsequent determination of the neutron energy spectrum. The possibility of using an iterative process for refining the results of the first approximation is investigated.

Let us consider the problem of determining the steady-state spatial and energy distribution of thermal neutrons in an elementary cell of a heterogeneous reactor. Although the distribution of neutrons with energies above some limiting value \( E_{\text{LI}} \) is of interest only in so far as it is needed to obtain a sufficiently accurate distribution for \( E < E_{\text{LI}} \), we shall formally seek a solution of the Boltzmann equation for neutrons of all energies. The resonance structure of the absorption cross section for \( E > E_{\text{LI}} \) need be taken into account only in the energy region close to \( E_{\text{LI}} \). In addition, since the spatial and energy distribution of sources (fission neutrons) does not affect the thermal group, the term describing the sources will be neglected as is usually done in determining thermal neutron spectra in an infinite homogeneous medium. We write the steady-state Boltzmann equation for the neutron density [1]:

\[
\frac{\Omega}{V} \frac{\partial n(E, \Omega, r)}{\partial t} + \Sigma_s(E, r)n(E, \Omega, r) = \int_0^\infty \frac{\sqrt{E}}{E} \Sigma_s(E' \to E, \cos \theta, r) \left[ n(E', \Omega', r') \right. \\
\left. \left\{ \int_0^{2\pi} d\phi \right\} - \Sigma_a(E, r)n(E, \Omega, r) \right].
\]

(1)

Here \( n(E, \Omega, r) \) is the number of neutrons per unit interval of phase space, \( \Sigma_s(E' \to E, \cos \theta, r) \) is the cross section for scattering through an angle \( \theta \) with a change in energy from \( E' \) to \( E \), \( \Sigma_s(E, r) \) and \( \Sigma_a(E, r) \) are the scattering and absorption cross sections, and \( \Omega \) and \( r \) are the angle and spatial variables.

Let us write \( \Sigma_s(E' \to E, \cos \theta, r) \) as the sum of an isotropic part \( \Sigma_s^{(i)}(E' \to E, \cos \theta, r) \) and an anisotropic part \( \Sigma_s^{(a)}(E' \to E, \cos \theta, r) \). The anisotropic part of the scattering cross section may be written approximately:

\[
\Sigma_s^{(a)}(E' \to E, \cos \theta, r) \approx \Sigma_s^{(a)}(E, \cos \theta, r) \delta(E - E'),
\]

(2)

where

\[
\Sigma_s^{(a)}(E, \cos \theta, r) = \int_0^{2\pi} \Sigma_s^{(a)}(E \to E', \cos \theta, r) d\phi.
\]

Let us write \( n(E, \Omega, r) \) as the sum of isotropic and anisotropic parts and rewrite Eq. (1) taking account of approximation (2)

\[
\frac{\Omega}{V} \frac{\partial n(E, \Omega, r)}{\partial t} + \Sigma_a(E, r)n(E, \Omega, r) = \int_0^\infty \Sigma_s^{(i)}(E, \cos \theta, r) n^{(a)}(E', \Omega', r') d\Omega' \\
- \Sigma_a(E, r)n^{(a)}(E, \Omega, r) + Q(E, r),
\]

(3)

where

\[
Q(E, r) = \frac{4\pi}{V} \int_0^\infty \sqrt{E'} \Sigma_s^{(i)}(E' \to E, r) \times n^{(a)}(E', \Omega', r') d\Omega' - \Sigma_a(E, r)n^{(a)}(E, \Omega, r).
\]

Equation (3) differs from the original Eq. (1) in the approximation made in the term determining the anisotropic scattering of neutrons belonging to the anisotropic component of the field:

\[
\int_0^\infty \sqrt{E'} \Sigma_s^{(i)}(E' \to E, \cos \theta, r) n^{(a)}(E', \Omega', r') d\Omega' \approx \int_0^\infty \Sigma_s^{(i)}(E, \cos \theta, r) n^{(a)}(E, \Omega, r) d\Omega'.
\]

(2')

To estimate the error introduced by this approximation we must first take into account the fact that the contribution of the approximate term is most important for scattering in hydrogen and also for an increase in the asymmetry of the flux, i.e., in very inhomogeneous systems. As a consequence of the equality of the integrals over energy on the left and right hand sides of the approximate Eq. (2), the effect of the approximation is significantly decreased in the solution of a problem in which the main interest is in quantities characterizing the slow neutron group as a whole.

A quantitative estimate of the error introduced by an approximation essentially equivalent to Eq. (2) was made by Honeck [2] for the special case of a uranium-water slab lattice. It was shown that the approximation hardly affects the disadvantage factor.

The energy dependence of the solution of Eq. (3) is determined solely by the integral operator appearing in \(Q(E, r)\). The other terms in the equation determine only the spatial and angular dependence of the neutrons of a given energy and the energy enters only as a parameter. It is important to emphasize that \(Q(E, r)\) does not depend on the angle variables since it describes the change in density due to the isotropic scattering of neutrons belonging to the isotropic component.

Taking these remarks into account we write \(n(E, \Omega, r)\) as a product of two functions

\[n(E, \Omega, r) = v(E) R_E(\Omega, r),\]

where \(v(E)\) is the number of neutrons in a cell per unit energy range and \(R_E(\Omega, r)\) is the spatial and angular distribution of the density of neutrons of energy \(E\). Since by definition

\[v(E) = \int \int n(E, \Omega, r) d\Omega dr,\]

the normalization condition for \(R_E(\Omega, r)\) is

\[\int \int R_E(\Omega, r) d\Omega dr = 1.\]

Substituting Eq. (4) into (3) and dividing by \(v(E)\) we obtain

\[\Omega \text{grad}_r R_E(\Omega, r) + \Sigma_s(E, r) R_E(\Omega, r) = \int \Sigma_s^{(a)}(E, \cos \theta, r) R_E^{(a)}(\Omega', r) d\Omega' - \Sigma_s(E, r) R_E^{(a)}(\Omega, r) + Q_E(r);\]

\[Q_E(r) = \frac{4\pi}{V_E v(E)} \int_0^\infty V_E^2 \Sigma_s^{(i)}(E' \rightarrow E, r) \times v(E') R_E^{(a)}(r) dE' - \Sigma_s(E, r) R_E^{(i)}(r),\]

where \(R_E^{(1)}\) and \(R_E^{(a)}(\Omega, r)\) are the isotropic and anisotropic parts of \(R_E(\Omega, r)\).

The set of equations (7) and (8) completely determines \(R_E(\Omega, r)\) and \(v(E)\), since it is equivalent to the original Eq. (1) to within the approximation (2). It is convenient, however, to use in addition the balance equation for \(v(E)\):

\[\Sigma(E) v(E) = \frac{1}{V_E} \int_0^\infty V_E^2 \Sigma_s(E' \rightarrow E) v(E') dE',\]

which can be obtained either directly from balance considerations or by integrating Eq. (7) over the angle and spatial variables. The bars over letters in Eq. (9) denote averages over the cell volume.

\[\overline{\Sigma}(E) = \frac{1}{V_E} \int_0^\infty V_E^2 \Sigma_s(E' \rightarrow E) v(E') dE',\]

\[\overline{\Sigma}_s(E' \rightarrow E) = \frac{1}{V_E} \int_0^\infty \Sigma_s^{(i)}(E' \rightarrow E, r) R_E^{(a)}(r) dr,\]

Let us consider the method of successive approximations for solving the system of Eqs. (7)-(10). If for any value of the energy the spatial dependence of the function \(Q_E(r)\) on the right hand side of Eq. (7)