Nonuniform multiple slot injection (suction) or
wall enthalpy into a steady compressible laminar
boundary layer

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Summary. An analysis is performed to study the effects of nonuniform multiple slot injection (suction) and nonuniform wall enthalpy on the steady nonsimilar compressible laminar boundary-layer flow over a two-dimensional and axisymmetric bodies. Nonsimilar solution is obtained from the origin of the streamwise coordinate to the exact point of separation. The difficulties arising out at the starting point of the streamwise coordinate, at the edges of the slots and at the point of separation are overcome by applying the method of quasilinear implicit finite difference scheme with an appropriate selection of finer step sizes along the streamwise direction. It is found that the nonuniform multiple slot injection moves the point of separation downstream but the nonuniform multiple slot suction has the reverse effect. Also, separation can be delayed more effectively by applying nonuniform multiple slot injection as compared to the nonuniform single slot injection. The nonuniform total enthalpy at the wall (i.e., the cooling or heating of the wall in the slots) along the streamwise coordinate has very little effect on the skin friction and thus on the point of separation.

1 Introduction

In recent years, a wide range of aerodynamic problems have come into prominence, due to the development of modern aircraft, guided missiles etc., that demands a detailed analysis of the flow in the boundary layer taking non-similarity into account. A review of non-similarity solution methods along with citations of relevant publications is given by Dewey and Gross [1]. Since then several investigators carried out research on the behavior of nonsimilar flow over two-dimensional and axisymmetric bodies both by approximate and exact methods such as local non-similarity method [2], asymptotic method [3], [4], differential-difference method [5], [6], momentum integral method [7] and finite difference method [8], [9].

In many cases of interest, the surface mass transfer (i.e., suction and injection) includes a tangential component at the surface and a component normal to the surface, thus constituting a vectored mass transfer [9]–[11]. Also, mass transfer through wall slot (i.e., mass transfer occurs in a small porous section of the body surface and there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for various potential applications including thermal protection, fuel injection in ramjet engines, energizing of the inner portion of boundary layers in adverse pressure gradients, and skin friction reduction on high speed aircraft. Moreover, mass transfer through slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region. Several investigators [12]–[16] have studied the effect of slot injection
(suction) into a laminar compressible boundary layer over a flat plate by taking the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuity at the leading and trailing edges of the slot. The discontinuities can be avoided by choosing a nonuniform mass transfer/wall temperature distributions in the slot [17], [18]. In a recent investigation, Roy and Nath [18] have studied the effects of nonuniform single slot injection (suction) and nonuniform total enthalpy at the wall into a steady non-similar compressible boundary-layer flow over two-dimensional and axisymmetric bodies.

The aim of this investigation is to study the effects of nonuniform multiple slot injection (suction) and nonuniform total enthalpy at the wall (wall cooling or heating takes place in the slots and the remaining part of the body surface has a constant value of the total enthalpy) on the steady non-similar compressible boundary-layer flow over two-dimensional body (cylinder) and axisymmetric body (sphere). Compressible boundary layer flow over two-dimensional or axisymmetric bodies provides many practical applications. For example, two-dimensional or axisymmetric bodies simulate approximately the leading edge of an aircraft or airplane body surface and also allow a basic simplification of the complicated three-dimensional compressible boundary-layer equations. Thus, the study of the effects of nonuniform multiple slot injection (suction) and nonuniform wall enthalpy on a steady non-similar laminar compressible boundary-layer flow over two-dimensional and axisymmetric bodies is useful in understanding many boundary-layer problems of practical importance as would rise, for example, for cooling gas turbine blades, for suppressing recirculating bubbles and for controlling transition and/or separation of the boundary layer over airplane control surfaces.

The nonsimilar solutions have been obtained starting from the origin of the streamwise coordinate to the point of separation (zero skin friction) taking nonuniform multiple slot injection (suction) or nonuniform wall enthalpy. The difficulties encounter at the origin of the streamwise coordinate, at the edges of the slots and near the point of separation have been overcome by using quasi-linearization technique with an implicit finite difference scheme. There are two types of free parameters in this problem, one type of parameters measures the length of the slots (i.e., the parts of the body surface in which total enthalpy has variations or there are mass transfer) and another type of parameters fixes the position of the slots. Thus, these two sets of parameters help to vary the lengths and locations of the slots.

It may be noted that the motivation came from author’s earlier investigation [18] by considering the recent demand of the numerical studies on the effect of surface mass transfer through multiple slots for the present day aerodynamic problems. The discontinuities at the leading and trailing edges of the slots have been avoided following [17], [18]. Thus, the present analysis differs from those in [12]–[16] with the finite discontinuities.

2 Basic equations and transformations

Consider a two-dimensional or an axisymmetric body of revolution in a compressible fluid. Let $x$ and $y$ be the curvilinear coordinates along and perpendicular to the boundary, respectively, $u$ and $v$ be the corresponding velocity components. The contour of the body of revolution is specified by the radii $r(x)$ of the section perpendicular to the axis (Fig. 1). The blowing rate is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the