Object Shape before Boundary Shape: Scale-Space Medial Axes*

STEPHEN M. PIZER, CHRISTINA A. BURBECK, JAMES M. COGGINS, DANIEL S. FRITSCH, AND BRYAN S. MORSE
Medical Image Display Research Group, University of North Carolina, Chapel Hill, NC 27599-3175

Abstract. Representing object shape in two or three dimensions has typically involved the description of the object boundary. This paper proposes a means for characterizing object structure and shape that avoids the need to find an explicit boundary. Rather, it operates directly from the image-intensity distribution in the object and its background, using operators that do indeed respond to "boundariness." It produces a sort of medial-axis description that recognizes that both axis location and object width must be defined according to a tolerance proportional to the object width. This generalized axis is called the multiscale medial axis because it is defined as a curve or set of curves in scale space. It has all of the advantages of the traditional medial axis: representation of protrusions and indentations in the object, decomposition of object-curvature and object-width properties, identification of visually opposite points of the object, incorporation of size constancy and orientation independence, and association of boundary-shape properties with medial locations. It also has significant new advantages: it does not require a predetermination of exactly what locations are included in the object, it provides gross descriptions that are stable against image detail, and it can be used to identify subobjects and regions of boundary detail and to characterize their shape properties.

Key words: shape description, object definition, multiscale method, medial axis, image analysis

1 Boundaries versus Medial Representations

The dominant train of thought in object shape measurement is based on boundary description. Thus for 2-D objects properties of the object edge, such as curvature, have been described, and for 3-D objects properties of the object surface, such as the loci of parabolic curves, flecnodal curves, gutterpoints, and ruffles [14], have received special attention. The difficulty of this approach is twofold. First, from the point of view of physics, for an object in an image there exists no edge locus without a tolerance since the object can exist only through imaging or visual measurements that have an associated spatial scale, and thus spatial tolerance [14], and the spatial scale that is appropriate for boundary definition is unclear. Second, shape involves certain global properties, which are not readily built into the process of describing boundaries. An important global property is that of involution, the relation between opposite points on two sides of an object (see figure 1 for examples).

Such global shape aspects can be captured more directly by focusing on the object middle-and-width combination that arises from pairing opposite object edges [2]. Blum proposed to do this by representing the object in terms of a

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Fig. 1. Involute: visually related opposite points on an object.
medial axis or skeleton running down the middle of the object, together with a width value at each point on the medial axis. His axis is defined such that for each axis point a disk centered at that point and with radius equal to the width value there is tangent to the boundary at two or more boundary points and is entirely within the object (figure 2). The endpoints of these central axes correspond to corners and other object boundary locations of locally maximal curvature [15], [16], the perceptual importance of which has long been known. It has also been noted [11], [12] that subjective edge perceptions derive especially strongly from high-curvature boundary points, such as line end and corners.

The width values \(w(s)\) of the middle/width representation carry straightforward access to the angle of the object boundary at each of the corresponding boundary points, relative to the axis direction at any axis point specified by arc length \(s: \theta = \cos^{-1}(dw/ds)\) [3]. Moreover, the curvature of the axis and of the boundary pair relative to the axis is also straightforwardly accessible. At axis endpoints the radii perpendicular to the boundary converge to a single boundary point, which is the visually important vertex of a protrusion, i.e., a relative maximum of boundary curvature. Axis branch points correspond to indentations in the object. Thus the middle/width representation incorporates major aspects of shape.

Blum also suggested a more general “global” form of the medial axis representation in which the multiply tangent disks need not be completely inside the object. Global axis sections for which the disks overlap the object’s background select boundary indentations and symmetries of larger width than the object, for example, the symmetry of the shorter sides of a rectangle.

The difficulty with Blum’s definition is that whereas it tackles the problem of global shape, it still requires an object boundary that is defined with zero tolerance. No method that requires such a boundary can be expected to be adequately insensitive to small-scale image properties, and, indeed, Blum’s method has been heavily criticized for this sensitivity.

2 Multiscale Geometry Detectors

Many investigators have suggested that notions of shape must be based on measurements in scale space, i.e., by sets of operators that sense a regional rather than curvilinear (e.g., edge or medial axis) property, with each operator sensing the same property but at different spatial scales. Among the operator kernels suggested have been derivatives of Gaussians [13], [19], differences of Gaussians [5], [26], Gabor functions [6], [23], Wigner operators [24], and wavelets [17], [18]. A persuasive argument for the form of operators, by ter Haar Romeny et al. [10], is that the system must be invariant to translation, rotation, and size change and that this implies multiscale operators \(h\) with kernels that are solutions to the diffusion equation \(\nabla \cdot [c(x; t)\nabla h(x; t)] = (\partial/\partial t)h(x; t)\), where \(t\) is half the square of the spatial scale \(\sigma\), \(x\) is a spatial location in \(\mathbb{R}^2\), and \(c\) is a conductance function that can vary in space and scale. Linear combinations of derivatives of a Gaussian with standard deviation \(\sigma\) satisfy this equation for \(c = 1\).

These operators, or combinations of them, can be thought of as giving the degree to which a point in scale space \(x, \sigma\) has the properties expected of a particular geometric feature. For example, we will say that boundariness is the degree to which the point behaves like a boundary and cornerness is the degree to which the point behaves like a corner. Similarly, we will say that medialness is the degree to which the point behaves like the middle of an object.

Boundariness at a particular location \(x\) and scale \(\sigma\) has typically been associated with variations in luminance about that location, i.e., with combinations of first or second partial