A NEW APPROACH TO THE PHENOMENOLOGICAL INTERPRETATION OF THE CONCEPT OF SLIPPAGE

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We propose a direct proof of the identity of the concepts of slippage and the Koiter version of the theory of plasticity based on parallel translation of planes enveloping the Tresk viscosity surface. One figure. Bibliography: 5 titles.

Koiter [1] has shown that the Batdorf-Budyanskii theory and the theory based on parallel translation of the planes enveloping the Tresk plasticity surface are identical. We shall give a new phenomenological interpretation of the concept of slippage. The new approach makes it possible to state relatively simple defining plasticity relations that reflect the physical mechanism of the process of plastic strain of a metal.

Consider the coincident Il'yuushin space of stresses and strains. Among the components of the stress vector $S_k$ ($k = 1, \ldots, 5$) and the components of the stress deviator $S_{ij}$ ($i, j = 1, 2, 3$) the following relations exist:

$$
S_1 = \frac{3}{2}(S_{zz} + S_{xx}); \quad S_2 = \frac{\sqrt{3}}{2}(S_{zz} - S_{xx}); \quad S_3 = \sqrt{3}S_{zz}; \quad S_4 = \sqrt{3}S_{yz}; \quad S_5 = \sqrt{3}S_{xy}.
$$

Accordingly the relations between the components of the strain deviator $e_{ij}$ and the components of the strain vector $e_k$ are defined by the relations

$$
e_{xx} = \frac{\varepsilon_1}{3} - \frac{\varepsilon_2}{\sqrt{3}}; \quad e_{yy} = -\frac{2}{3}\varepsilon_1; \quad e_{zz} = \frac{\varepsilon_1}{3} + \frac{\varepsilon_2}{\sqrt{3}}; \quad e_{xz} = \frac{\varepsilon_3}{\sqrt{3}}; \quad e_{yz} = \frac{\varepsilon_4}{\sqrt{3}}; \quad e_{xy} = \frac{\varepsilon_5}{\sqrt{3}}.
$$

In the context of the theory of slippage the beginning of plastic strain is connected with the fulfillment of the Tresk-Saint-Venant condition. The Tresk viscosity condition $\tau_{\text{max}} = \tau_s$ in the six-dimensional space of components of the stress tensor corresponds to a plasticity surface whose equation is of a complicated form. For the concept of slippage this surface can be represented as the envelope of a three-parameter family of planes $\sigma_{ij}l_in_j = \frac{\sigma_s}{\sqrt{3}}$ [1]. Here $\sigma_{ij}$ are the components of the stress tensor, $\sigma_s$ is the yield point under elongation; $l_i$ and $n_j$ are the direction cosines of the surface with normal $n$ given by angles $\alpha$ and $\beta$ and the direction of slippage $l$ in this plane determined by angle $\omega$. Such planes to not encompass all possible orientations of planes of the six-dimensional space, but they do determine uniquely an arbitrary slippage system $n, l$ in which a shear strain can arise. Consequently the Tresk viscosity surface in the six-dimensional space of components of the stress tensor is completely determined by the three-dimensional subspace generated by the space of Euler angles $\alpha, \beta$, and $\omega$.

In analogy with the Tresk viscosity state we write the plasticity condition $\sigma_{ij}l_in_j = \sigma_s/\sqrt{3}$ in terms of the components of the stress vector $S_k$ of (1). After certain transformations we obtain

$$
S_1\sqrt{(n_xl_x + n_zl_z)} + S_2(n_xl_z - n_zl_x) + S_3(n_xl_x + n_zl_z) + S_4(n_xl_y + n_yl_x) + S_5(n_yl_z + n_zl_y) = \sigma_s.
$$

To this plasticity condition in five-dimensional space there corresponds the envelope of the planes (3) all lying at the distance $\sigma_s$ from the origin.

Writing the viscosity conditions in a form similar to (3) leads to the conclusion that the Batdorf-Budyanskii theory and the theory of plasticity for materials with a singular load surface [1] coincide. Let us examine this proof in the new interpretation.

In the five-dimensional space of components of the stress vector we introduce a unit radius vector perpendicular to the plane

\[
\mathbf{r} = \sqrt{3}(n_z l_z + n_z l_x)\mathbf{e}_1 + (n_z l_z - n_z l_x)\mathbf{e}_2 + (n_x l_z + n_z l_x)\mathbf{e}_3 + (n_z l_y + n_y l_x)\mathbf{e}_4 + (n_y l_z + n_z l_y)\mathbf{e}_5,
\]

where \(\mathbf{e}_k\) \((k = 1, \ldots, 5)\) are unit orthonormal basis vectors.

We define the infinitesimal vectors \(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3:\)

\[
\mathbf{r}_1 = \frac{d\mathbf{r}}{d\alpha} d\alpha, \quad \mathbf{r}_2 = \frac{d\mathbf{r}}{d\beta} d\beta, \quad \mathbf{r}_3 = \frac{d\mathbf{r}}{d\omega} d\omega.
\]

In five-dimensional space consider the polyvector \(d\mathbf{V}\) represented by the vector product of the vectors (5) [2] (see figure)

\[
d\mathbf{V} = [\mathbf{r}_1 \times \mathbf{r}_2 \times \mathbf{r}_3].\]

All the normals to the planes (3) passing through the three-dimensional volume element of the parallelepiped constructed on the vectors \(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\) lie in the subspace generated by the polyvector \(d\mathbf{V}\).

The square of the unknown three-dimensional volume is given by the Gram determinant of the vectors \(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3\) [2]:

\[
(d\mathbf{V})^2 = \det \| (\mathbf{r}_i, \mathbf{r}_j) \| \quad (i, j = 1, 2, 3),
\]

where \((\mathbf{r}_i, \mathbf{r}_j)\) is the scalar product of the vectors \(\mathbf{r}_i\) and \(\mathbf{r}_j\).

Taking account of (5), after suitable transformations we obtain

\[
d\mathbf{V} = 2 \cos \beta d\alpha d\beta d\omega.
\]

The relative number of planes (3) with normals inside the volume \(d\mathbf{V}\) determined by the polyvector \(d\mathbf{V}\) is proportional to \(d\mathbf{V}\). It is assumed that in the loading process the planes (3) in five-dimensional space are translated parallel to themselves from the origin. The motion of the planes causes an increment in the plastic strain depending on the size of the displacement and directed along the normal to the moving plane. The unit increment in the plastic strain from the motion of the plane with normal \(\mathbf{N}\) is

\[
0 \gamma^0_N = F(H_N),
\]

where \(F\) is the characteristic function of the material and \(H_N\) is the distance from the origin to the plane with normal \(\mathbf{N}\).

The plastic strain of the system of planes (3) with normals inside an oriented volume element is proportional to \(d\mathbf{V}\),

\[
d\gamma^p_N = F(H_N) d\mathbf{V}.
\]

The components of the plastic strain (10) with respect to the axis of five-dimensional Euclidean space can be written in the form

\[
d\varepsilon^p_k = N_k d\gamma^p_N,
\]

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