ANALYTIC PROPERTIES OF THE SCATTERING MATRIX
OF MANY PARTICLE SYSTEMS

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Dedicated to the dear memory of David Milman

We generalize the previous results on a meromorphic continuation of the scattering matrix for $N$ particle systems. Our proofs seem to be much simpler than those given before.

INTRODUCTION

The scattering matrix is the central object in the scattering theory. It maps the scattering data at the distant past into the data in the remote future. Mathematically, it is an operator-valued function, $S(E)$, depending on the total energy $E$ of the system in question. It is conjectured on basis of an analysis of simple examples that it is meromorphic function of $\sqrt{E}$ with the poles occurring exactly at the bound state energies (eigenvalues) and resonances of the underlying Schrödinger operator $H$. In fact, the physicists use this conjecture (or the related conjecture about the scattering cross-section) to define the quantum resonances.

The above conjecture is proven completely only for a special subclass of the class of one-body systems. For the many-body systems there are only very partial results (which the present paper generalizes to a certain extend). The reason for such a slow progress is that, apart from the special 2-cluster→2-cluster scattering processes, the proof of meromorphic continuation involves difficult resolvent estimates (see Sigal, 1983). To complicate the matter further the above conjecture without qualifications is not true in the many-body case: in general, whenever $S(E)$ has a meromorphic continuation
it is realized on a Riemannian surface more complicated than that of the function $\sqrt{Z}$ as originally conjectured (see the recent work of Derezinski, 1985).

Now we turn to the notion of resonance. Clearly, the physicist’s definition of resonances is a handicap. Fortunately, there is another definition, introduced in Mathematical Physics (Balslev and Combes 1971, Simon, 1973), stated directly in terms of the original Schrödinger operator (or more precisely, in terms of a family of Schrödinger operators related to the original one) and which is applicable to a considerable class of many-body systems. This definition is very close to the definition of the bound states and the resonances defined in this way will be called the spectral resonances. This leads us to the problem of identification of the poles of a meromorphic continuation of the scattering matrix with the spectral resonances.

The results on meromorphic continuation of the scattering matrix and identification of its poles in the many-body case are due to Balslev 1980 a,b (three particle scattering), Hagedorn 1979 (four particle, 2-cluster → 2 - cluster scattering), Sigal 1981, 1982 (N particle, single channel scattering). The present paper generalizes these results considering N particle systems and the scattering processes for which either one of the channels (incoming or outgoing) is two-clusters or both channels have the same internal energy. (This exhausts all the possibilities in the case of N = 3). Besides, our proofs seem to be much simpler than the proofs of the papers mentioned above. Independently, Derezinski, 1985, has generalized the result of Hagedorn, 1979 to N particle systems, with the energy below the lowest three-cluster threshold, and more general potentials. Besides, unlike the previous authors who considered only a neighbourhood of the real axis, Derezinski proves the existence of a meromorphic continuation into the entire piece of a Riemannian surface defined by the above energy condition.