On the Contributions of John C. Harsanyi, John F. Nash and Reinhard Selten

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The Royal Swedish Academy of Sciences awarded the 1994 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel to John C. Harsanyi, John F. Nash and Reinhard Selten for “their pioneering analysis of equilibria in the theory of non-cooperative games”. The Academy justifies giving the prize in economics to three game theorists by the argument that strategic interaction characterizes many economic situations and that, therefore, game theory has proved very useful in economic analysis. Today, 50 years after the publication of Von Neumann and Morgenstern’s Theory of Games and Economic Behavior (1944), game theory, and, in particular, its non-cooperative branch has become a dominant tool for analyzing economic issues. In this note, I briefly review and appraise the laureates’ fundamental contributions to game theory and the economic sciences.

1 Equilibria in Non-Cooperative Games

The principal aspect of non-cooperative game theory is the equilibrium concept introduced by John Nash in his PhD thesis: “Non-cooperative Games” (Nash (1950b)). The thesis also introduces the fundamental distinction between cooperative and non-cooperative games. In games of the latter type, players are unable to conclude enforceable agreements outside the formal rules of the game. As Nash puts it “The basic requirement for a non-cooperative game is that there should be no-preplay communication among the players [unless it has no bearing on the game]. Thus, by implication, there are no coalitions and no side-payments” (Nash 1950b, p. 21).

Nash’s equilibrium concept is a generalization of the minimax solution introduced in Von Neumann (1928) for the two-person zero-sum game. Nash’s main mathematical result is the proof of existence in any finite, strategic form game, of at least one Nash equilibrium. In published work (Nash (1950a, 1951)), Nash provides two alternative, elegant proofs, one based on Kakutani’s fixed point theorem, the other based directly on Brouwer’s theorem. These techniques have inspired many other existence proofs, for example, in the area of general equilibrium theory. (See Debreu (1984).)

In the section “Motivation and Interpretation” of his thesis, Nash discusses two interpretations of his equilibrium concept. In the first, “mass-action” interpretation of
equilibria “It is unnecessary to assume that the participants have full knowledge of
the structure of the game, or the ability and inclination to go through any complex
reasoning processes. But the participants are supposed to accumulate empirical in-
formation on the relative advantages of the various pure strategies at their disposal”
(Nash 1950b, p. 21). If the frequencies with which the various strategies are played
settle down, then these stable frequencies must constitute an equilibrium. As Nash
notes, the interest in this interpretation derives from the fact that “There are situations
in economics or international politics in which, effectively, a group of interests are
involved in a non-cooperative game without being aware of it, the non-awareness
helping to make the game truly non-cooperative” (Nash 1950b, p. 23).

In contrast, the second (and more familiar) “rationalistic interpretation” requires
that the players are rational and know the full structure of the game. It originates from
considering the question: “What would be a “rational” prediction of the behavior to be
expected of rational playing the game in question?” (Nash 1950b, p. 23). A theory of
rational behavior that provides a unique strategy recommendation for each player has
to prescribe to play a Nash equilibrium, since otherwise it is not immune to “theory
absorption”, i.e. it is self-destroying. Of course, this argument does not imply that any
theory that prescribes to play a Nash equilibrium is a satisfactory one, nor that such
a satisfactory theory necessarily exists. (See von Neumann and Morgenstern (1944,
Sects. 4.2, 4.3 and 17.3). In particular the founding fathers insist that the solution
from this indirect argument be independently justified by a direct argument.)

Since the uniqueness assumption is crucial for the rationalistic justification2 of
Nash’s equilibrium concept, the fact that a game frequently has multiple equilibrium
points is problematic for that interpretation. In order to cope with this multiplicity,
Nash introduces several auxiliary concepts (such as interchangeability, solutions and
sub-solutions), however, many games do not admit solutions. Nevertheless, Nash ar-

gues that it sometimes happens that good heuristic reasons can be given for narrowing
down the set of equilibria to a single solution. In particular, in an important applica-
tion to the study of cooperative games, Nash (1953) suggests to discriminate among
equilibria by studying their relative stabilities. Specifically, he perturbs a game and
studies the limiting behavior of the associated equilibria as the perturbations vanish.
Nash shows that the unique necessary limit is the outcome that he had derived
earlier by means of the axiomatic method (Nash (1950c)). Nash’s suggestion to cut
down on the number of equilibria in this way has been extensively followed in the
literature.

Nash argues that game theory’s cooperative (axiomatic) and non-cooperative ap-
proaches are complementary, that each helps to justify and clarify the other. If a
solution can be obtained from a convincing set of axioms, this indicates that the solu-
tion might be appropriate in a wider variety of situations than those formally captured
by a specific non-cooperative model. On the other hand, by following the so-called
Nash program, i.e. by reformulating cooperative games as non-cooperative ones and
by solving for the Nash equilibria of the latter, an abstract discussion about the rea-

2 In the remainder, attention will be confined to this interpretation. Recently, the mass-action inter-
pretation has regained attention. For a partial review of recent developments that build on it, the
reader is referred to Weibull (1994).