BENDING VIBRATIONS OF BIMORPH PIEZOCERAMIC PLATES

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Equations are derived for bending vibrations of bimorph piezoceramic plates allowing for the type of polarization of the layers and the conditions of application of electric power to electrode surfaces. Boundary-value conditions with open electrodes are considered.

We develop the main relationships of the applied theory of bending vibrations of thin bimorph piezoceramic plates. Their derivation is based on the Kirchhoff–Love classical conjectures for the mechanical variables of the conjugate field and the complementary conjectures for the electrical variables. Note that the theory of deformation of thin-walled piezoelements [2, 3] does not consider the theoretical principles of calculation of bimorph plates. Yet bimorph plates are a standard component in many measuring systems.

1. Bending Vibrations of Rectangular Bimorph Plates

Consider a piezoceramic bimorph plate of thickness h (Fig. 1). The points of the plate are defined by the Cartesian coordinates x, y, z. The origin is at the middle plane of the plate, and the Oz axis is perpendicular to the middle plane. A bimorph plate consists of two rigidly joined plates of thickness h/2 coated by electrodes on opposite sides. We assume that the contour surface S of the plates is free from electrode coating. Bending of the plate is produced not only by normal loads applied to its midplane, but also by an electric potential difference across the electrodes.

Using Kirchhoff's static conjectures, we assume that the normal and tangential stresses on surface elements parallel to the midplane are zero, i.e.,

$$
\sigma_z = 0; \quad \tau_{zx} = 0; \quad \tau_{zy} = 0.
$$

For the electric variables of the conjugate field with deposited electrodes, the analogs of Kirchhoff's conjectures for the tangential components of the electric field and the induction are [1]

$$
E_x = 0; \quad E_y = 0; \quad D_x = 0; \quad D_y = 0.
$$

In accordance with Kirchhoff's generalized conjectures, the equations of state for bimorph rectangular plates are written in the form [1]

$$
\begin{align}
\varepsilon_x &= S_{11}^{f} \sigma_x + S_{12}^{f} \sigma_y \pm d_{33} E_z; \\
\varepsilon_y &= S_{12}^{f} \sigma_x + S_{11}^{f} \sigma_y \pm d_{33} E_z; \\
\varepsilon_z &= S_{13}^{f}(\sigma_x + \sigma_y) \pm d_{33} E_z; \\
\varepsilon_{xy} &= 2(S_{11}^{f} - S_{12}^{f}) \tau_{xy}; \\
D_x &= \varepsilon_{33} E_z \pm d_{33}(\sigma_x + \sigma_y);
\end{align}
$$


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Fig. 1. Bimorph rectangular piezoceramic plate in a generator circuit.

where $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$, $\varepsilon_{xy}$ are the strain components; $\sigma_x$, $\sigma_y$, $\tau_{xy}$ are the components of the mechanical stress tensor; $E_z$ is the component of the electric field vector $\vec{E}$; $D_z$ is the component of the induction vector $\vec{D}$; $S_{11}^E$, $S_{12}^E$, $S_{13}^E$ are the elastic compliances for zero electric field; $d_{31}$, $d_{33}$ are the piezoelectric constants; $\varepsilon_{33}^T$ is the dielectric permittivity for zero stress.

The top sign of $d_{31}$ corresponds to the bottom plate and the bottom sign to the top plate (see Fig. 1).

The system of equations (1.3) can be rewritten in the form

$$
\sigma_x = \frac{1}{S_{11}^E(1 - \nu^2)}[(\varepsilon_x + \nu\varepsilon_y) + (1 + \nu)d_{31}E_z];
$$

$$
\sigma_y = \frac{1}{S_{11}^E(1 - \nu^2)}[(\nu\varepsilon_x + \varepsilon_y) + (1 + \nu)d_{31}E_z];
$$

$$
\tau_{xy} = \frac{1}{2(S_{11}^E - S_{12}^E)}\varepsilon_{xy};
$$

$$
D_z = \varepsilon_{33}^T(1 - K_p^2)E_z \pm \frac{d_{33}}{S_{11}^E(1 - \nu)}(\varepsilon_x + \varepsilon_y),
$$

where $\nu = -S_{12}^E/S_{11}^E$, $K_p^2 = 2d_{31}^2/(1 - \nu)\varepsilon_{33}^T S_{11}^E$ are the Poisson ratio in the isotropy plane $xOy$ and the static planar electromechanical coupling coefficient.

The main variables in (1.3) are identified on the basis of Kirchhoff’s kinematic conjecture, which establishes a relationship between the planar components of the displacement vector and the transverse displacements of the midplane points of the plate [4]:

$$
u_x = -z\partial W/\partial x; \quad \nu_y = -z\partial W/\partial y.
$$

From the expressions for displacements (1.5) and the Cauchy relationships

$$
\varepsilon_x = \frac{\partial u_x}{\partial x}; \quad \varepsilon_y = \frac{\partial u_y}{\partial y}; \quad \varepsilon_{xy} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y},
$$

we obtain the strains

$$
\varepsilon_x = -z \frac{\partial^2 W}{\partial x^2}; \quad \varepsilon_y = -z \frac{\partial^2 W}{\partial y^2}; \quad \varepsilon_{xy} = -2z \frac{\partial^2 W}{\partial x \partial y},
$$

From the conditions of joint deformation of the two plates it follows that each plate experiences both planar and bending strain.

By the conjectures for the electrical variables of the conjugate field [1], the distribution of the normal components of the electric field and the induction across the plate is approximately represented by the equalities

$$
E_z = E_0 \pm z\varepsilon_{33}; \quad D_z = D_0,
$$

where $E_0$, $E_1$, and $D_0$ are independent of the coordinate $z$. The choice of the sign in the first equality is determined by the direction of preliminary polarization (plus for the bottom plate, minus for the top plate).

By the standard rule, we introduce the electrostatic potential $\vec{E} = -\nabla \psi$. In our case,

$$
E_z = \partial \psi / \partial z.
$$