Oscillatory instabilities of the liquid and mushy layers during solidification of alloys under rotational constraint

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Summary. Linear flow instabilities due to oscillatory disturbances of the liquid and mushy regions during solidification of binary alloys are investigated under a rotational constraint where the rotation axis is inclined to gravity vector. Results of stability analyses and numerical computations for a preferred centrifugal mode of general oscillatory disturbances at zero and non-zero rotation rates are determined which provide information about the preference of oscillatory flow and its role on the solidification system as modified by the rotational effects. The main results are due to a preferred oscillatory mode of convection which is more significant for non-zero rotation case and is restricted mostly to the mushy region. The preferred oscillatory mode of convection is a traveling wave in the presence of rotation, but it is a standing wave in the absence of rotation. The results for different Prandtl numbers indicate that the freckles formation tendency for metallic alloys is less than that for aqueous solutions. Freckles are imperfections that reduce the quality of the solidified materials.

1 Introduction

Recently Worster [1] investigated instabilities of the liquid and mushy regions during unidirectional solidification of alloys and in the absence of any rotational effects. He studied only the onset of non-oscillatory instabilities. He stated in his paper [1] that oscillatory instabilities in systems in which lower diffusing component is unstably stratified, such as his system, usually give way to direct modes of instability that lead to the formation of double-diffusive fingers. Accordingly, he superimposed non-oscillatory disturbances on the base flow and performed a linear stability analysis of the problem. Most of his results are for the cases of aqueous solutions whose representative Prandtl number Pr is Pr = 10, while the rest of his results are for the cases of metallic alloys whose appropriate Prandtl number is P = 0.02. Worster [1] discovered two modes of convection and called them appropriately the mushy layer mode and the boundary layer mode. The mushy layer mode is driven by buoyant residual fluid within the mushy layer of dendrite crystals, while the boundary layer mode is associated with a thin compositional boundary layer in the melt just above the mush-liquid interface. Depending on the parameter values, due to thermodynamic and physical properties of the alloy, either one of these modes can dominate. Worster [1] determined various results including marginal stability curves for different parameter values and found good agreement between his results of linear stability analysis for Pr = 10 and the experimental results of Tait and Jaupart [2] for the onset of the mushy layer mode of convection in aqueous solutions of ammonium chloride.

More recently Sayre and Riahi [3] investigated non-oscillatory instabilities of the liquid and mushy regions during solidification of binary alloys under a high gravity environment, where it was assumed that the solidification system was placed in a centrifuge basket [4] whose rotation...
axis was inclined with respect to the high gravity vector. The high gravity vector was also assumed to be anti-parallel to the direction of the solidification growth rate. They considered a linear stability system due to two-dimensional non-oscillatory disturbances and developed a numerical code which was used to determine the results for various flow features due to the preferred stationary instability modes of the stability system. Their results indicated two preferred distinct modes of stationary convection whose characteristics are no longer of the kind detected by Worster [1] since rotational effects altered significantly the form of these modes. They appropriately called these two modes short wavelength and long wavelength modes since they had distinct short and long wavelength characteristics which persisted even in the case of strong rotation. They found that rotational effects made both of these stationary modes more dependent on the internal structure of the mushy layer and resulted in the production of negative perturbations of the solid fraction within the mushy layer that are indicative of freckle formation tendencies, although the long wavelength mode was found to be more effective in such production processes. The spatial locations in the mushy region which tended to form freckles were found to change as the rotation rate decreased or increased. This led the authors to suggest a controlling procedure for the possible elimination of freckles, namely application of a variable rotation rate.

In the present investigation, we consider the same marginal stability problem as the one studied earlier [3], but for the oscillatory disturbances, and for the case where the solidification growth rate direction is anti-parallel to the normal gravity vector and is inclined to the rotation axis. This system turns out to be mathematically the same as the high gravity system treated before by the present authors [3], provided that the Rayleigh numbers are defined based on the acceleration due to normal gravity \( g \) and not based on the magnitude of the high gravity vector. An interesting result of the present study, in contrast to the ones due to stationary instability cases [1], [3], is the persistence of a preferred oscillatory mode which is favored more by rotational constraint, by the metallic alloys and by the mushy region.

2 Formulation

We consider a thin layer of a binary alloy melt of some constant composition \( C_o \) and temperature \( T_o \) which is solidified at a constant rate \( V_o \) with the eutectic temperature \( T_e \) at the \( z = 0 \) held fixed in a frame moving with the solidification speed in the vertical \( z \)-direction (anti-parallel to the gravity vector) (Fig. 1). The physical model is based on the assumptions of the type considered by