Note

Rotations: Matching Schemes that Efficiently Preserve the Best Reply Structure of a One Shot Game

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Abstract: Suppose that we have a two player game in which we want to test experimentally whether the subjects learn to play the game theoretic solution. For this purpose we need a matching scheme which assures that a rational subject behaves in each round of the experiment as if he played a separate stage game. In this paper we show that such a 'best-reply-structure-preserving matching scheme' has to be free of repercussion effects, and that the rotation of two equally sized groups of subjects, which was introduced by Cooper, DeJong, Forsythe and Ross, solves the problem efficiently.

Keywords: Best Reply Structure, Repeated Games, Experimental Design, Matching.

1 Introduction

In decision problems it is not difficult to test the hypothesis that subjects learn to behave according to a theoretical prediction. The experimenter has to confront the subjects with the same decision problem several times and then he can test his hypothesis with a statistical analysis of the decision data. If the same question is raised in a simple game model this straightforward approach is not so easy to implement. Most standard experimental designs create a potential for strategic interdependencies between the rounds of the session so that game theory requires an analysis of the whole experimental session, and it often turns out that the theoretical solution does not allow us to interpret the observed individual decisions as a sequence of actions chosen in a sequence of separate stage games.

In this paper I will address the question how the matching should be organized if one wants to avoid this problem in an experiment on two-player stage games. This question is closely related to the problem analyzed by Kandori [1992]. Kandori discussed how much, or better, how little information about the past is needed to enforce cooperative play in a community in which the members play a repeated prisoner's dilemma game against anonymous changing partners. To illustrate his argument Kandori introduced what he calls 'contagious strategies'

\[1\] A prisoner's dilemma is a symmetric game with two actions called 'defect' and 'cooperate' in which 'defect' dominates 'cooperate' while the outcome in which both players cooperate Pareto-dominates the outcome in which both players defect.

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in which the players cooperate whenever they met a cooperating opponent in the previous round and defect forever after the first defection. In an infinitely repeated prisoner’s dilemma among changing members of a finite population these strategies enforce a cooperative equilibrium if the discount factor is high enough, because a player who defects will eventually lose his short run gain as he will play infinitely often against a defecting opponent. Thus Kandori shows that community enforcement of cooperative play is possible without information about the partner or the matching procedure.

In order to answer the question raised above I have to reverse Kandori’s argument to show how the subjects in an experiment can be assigned to each other without introducing the slightest possibility of such a community enforcement effect into the repeated game. In Kandori’s example every member of the finite population meets at least one opponent infinitely often so that the fear of this opponent’s reaction is sufficient to discipline a patient player. But if one wants to preserve the best reply structure of the stage game it is not enough to avoid this simple punishment mechanism. To guarantee that a rational player should behave in each round as if he solves a separate strategic problem it is also necessary to exclude all repercussion effects so that there is no possibility that a decision of a participant can influence the decision of one of his future opponents.

In section 3 I will show that the following rotation of two equal sized groups maximizes the number of observed individual decisions among all such best-reply-structure-preserving matching schemes: the \( N \) subjects who participate in each session are partitioned into two groups of approximately equal size, and the \( N_1 \) members of one group (the larger if there is one) are ordered on a circle; in the first matching each member of the other (smaller) group is assigned to one member of the ordered group and in the \( N_1 - 1 \) following matchings each member of the unordered group moves one step on the circle to meet his next partner.

The rotation of two equal sized groups was first used by Cooper, DeJong, Forsythe and Ross [1995] to generate a sequence of one shot prisoner’s dilemma games without reputation effects. Unfortunately their paper is a little bit misleading about the merits of the experimental design. Refering to Kandori’s terminology Cooper et al. say that the rotation has a ‘no-contagion’ property which implies that 40 participants are enough to generate 800 decisions in a one stage prisoner’s dilemma game, but they forget to mention that these observations are not necessarily independent so that one might get the impression that this large set of observations is obtained at no cost. If every participant in this experiment uses Kandori’s contagious strategies, a first round defection of one subject leads to a defection of the whole group in the last round so that the behavior of one subject can still contaminate the behavior of the whole group. The number of independent observations in this experiment

\[ p. 11-13. \text{In an earlier [1991] version of the paper the explanation of the merits is a little bit longer but not more careful.} \]

\[ As \text{Davis and Holt [1993, p.528] point out, this problem appears whenever the subjects use history dependent strategies.} \]