Decay of a Discontinuity Line in a Viscous Fluid

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With 21 Figures

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Summary. Integral transforms are used to solve singular initial value problems for STOKES slow-motion equations. The method is applied to investigate the decaying process of straight and circular discontinuity lines as well as the dissipation of local disturbances in an infinite medium. A criterion for the occurrence of secondary vortices is derived. Numerical results are displayed for periodic initial disturbances which demonstrate graphically the spreading of the disturbance from a discontinuity line into the fluid under successive development and decay of secondary vortices. A more detailed sequence of dissipating vortices is evaluated numerically and displayed by streamline patterns in connection with LAMB's vortical eigenmotions in an infinitely long cylinder of finite radius.


I. Introduction

In a recent paper [1] complete systems of time-dependent separable solutions of STOKES' slow-motion equations have been constructed for rectangular and annular flow regions. This was achieved by means of a generalized separation technique introduced in [2, 3]. In addition, so-called associated separable solutions have been found by differentiating or integrating separable solutions with respect to an integration parameter. Their significance for the construction of certain solutions of the complete NAVIER-STOKES equations has been pointed out.

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With the aid of those separable solutions eigenmotions have been determined which are required to satisfy certain incomplete boundary data without prescribed initial values. For given boundary and initial data a discrete or continuous superposition of eigenmotions is necessary. Because of the complicated nature of this fitting procedure the introduction of integral transforms suggests itself. They can be utilized because separable solutions exist.

In this paper the integral transform technique is applied to describe decaying straight and coaxial discontinuity lines as well as the obliteration of local disturbances in a laminar fluid flow. The analysis is restricted to plane motions so that use can be made of the stream function $\Psi$. Hence, Stokes’ slow-motion equations are reduced to the single fourth order partial differential equation

$$\left( \Delta - \frac{\partial}{\partial t} \right) \Delta \Psi = 0,$$

where $\Delta$ is the Laplacian operator and $t$ the product of time and constant kinematic viscosity. Numerical results are obtained for selected examples which demonstrate graphically the sequence of decay.

It may be mentioned that under certain boundary and regularity conditions the solutions obtained can be used to derive integrals of the quasilinear Navier-Stokes equations in an exact linear manner by an iterative procedure. This has been shown in [1, 2, 3] for various flow problems.

II. Decay of a Straight Discontinuity Line in an Infinite Region

A straight discontinuity line may be generated by two uniform parallel flows of opposite direction at $t = 0$ and may be located at the axis $y = 0$ in a Cartesian coordinate system $(x, y)$ (Fig. 1). In addition, a disturbance along the discontinuity line is superposed which may be of periodic or aperiodic nature expressed by a Fourier series or Fourier integral, respectively. Hence, a slow-motion solution of Stokes’ equation (1) in Cartesian coordinates

$$\nabla \Psi_{xxxx} + 2 \nabla \Psi_{xxyy} + \nabla \Psi_{yyyy} = (\nabla \Psi_{xx} + \nabla \Psi_{yy}) t$$

(2)